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Multipurpose modified iterative solver for nonlinear equations

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ABSTRACT

Non-linear Eq.s occur as a sub-problem in a wide variety of engineering and scientific domains. To deal with the complexity of Non-linear Eq.s, it is often required to use numerical procedures, which are the most suitable method to employ in certain circumstances. Many classic iterative approaches have been regularly employed for various situations; nevertheless, the convergence rate of those methods is low. In many cases, an iterative approach with a faster convergence rate is needed. This is something that classical methods like the Newton-Raphson Method (NRM) cannot provide. As part of this investigation, a modification to the NRM has been suggested to speed up convergence rates and reduce computational time. Ultimately, this research aims to improve the NRM, resulting in a Modified Iterative Method (MIM). The proposed method was thoroughly examined. According to the research, the convergence rate is higher than that of NRM. The proposed method delivers more accurate results while reducing computational time and requiring fewer iterations than earlier methods. The numerical findings confirm that the promised performance is correct. The results include the number of iterations, residuals, and computing time. This innovative technique, which is appropriate to any Non-linear equation, produces more accurate approximations with less iteration than conventional methods, and it is appropriate to any Non-linear equation.

1. Introduction

Determining the roots of a Non-linear Eq. is one of the most frequent difficulties faced in applied mathematics. Graphical approaches are either incorrect or use substantial memory [1]. The Bisection Method (BM), the Regula Falsi Method (RFM), the Secant Method (SM), and the NRM are examples of frequently used algorithms that are simple to comprehend [2].

The most difficult difficulty in numerical analysis is to describe Non-linear Eq.s, which are widespread in the field of engineering and can occur in a variety of ranges [1]. The demonstration characteristics and convergence can be exceedingly complex when compared to the initial hypothesis of the outcome [2]. A significant deal of effort has gone into understanding the structure of Non-linear Eq.s, and many useful models and methods have been proposed to do so [5].

Several approaches, including newer blends of traditional numerical methodologies and smart processes, are effective in explaining the system of Non-linear Eq.s, which are overwhelmed by the difficulties of making a reasonable first guesstimate of the outcome [3]. Even still, selecting a plausible primary premise of explanation for the vast majority of systems of Non-linear Eq. is extremely difficult to accomplish [4]. More than that, the computational efficacy isn't particularly impressive [5]. It is possible that the rate of convergence to the primary prediction of the explanation for furthermost numerical processes, such as the NRM is significantly more complicated when compared to the primary prediction [8]. It is, nevertheless, highly challenging to select a workable fundamental postulate of the conclusion for the system of Non-linear Eq.s that is consistent with the remainder of the system [9].

The Eq. in which the term has a power of two or more than two can be utilized in the case of a Non-linear Eq. in order to solve it. [6]. A curve is formed on a graph by Non-linear Eq.s, and as the power of the term rises, the graph's curve expands in size and changes its direction continuously. The graph depicts differences in slope at a variety of locations [7]. According to generalized linear Eq. theory, the Non-linear Eq. has the form $ax^2 + by^2 = c$, wherever x and y are the variables and a, b, and c are the constant values. Non-linear Eq.s systems typically have numerous solutions, no solutions, or a single solution in most cases [8]. At the solution point of a Non-linear system, there exist several Eq.s that are similar to the number of variables, and each of these Eq.s is satisfied [9].

An extensive array of applications for non-linear systems can be initiate in the physiology of nerves, chemical reactions and turbulence, electrical circuits, cardiac regulation, secure communications and encryption, celestial mechanics, economics and population increase, to name of few examples [10].

The so-called predator-prey or Lotka Volterra system is a simple Non-linear systems application that can be found in many places [11]. When the systems emerge, two species may be linked together, one of which will be prey and the other which will be predator [12]. When it comes to biology, a system of Eq.s is used to describe the natural periodic variations of populations of various types [13]. It is traditionally difficult to solve Non-linear Eq.s precisely, and exact solutions are only found very infrequently. For this reason, iterative methods are employed in order to find solutions to these Eq.s. [14].

2. Methodology

The purpose of this study is to evaluate whether the convergence rate increases as a result of improvements to currently available approaches. Also considered is the impact of the convergence rate, whether it is slow or quick. It also analyses the convergence rate of different approaches, and on the basis of the analysis of the convergence rate, a change will be evaluated that takes advantage of the advantageous qualities of the different methods. The major approach taken in this work will be the application of the MIM to a set of randomly generated Non-linear Eq.s, with the accuracy and computational time being investigated. The Matlab programming language is used to implement the MIM described above. Trigonometric, exponential, logarithmic, and cubic polynomial functions are examined using the Modified Iterative Programs (MIP).

1.1 Modified Iterative Model

Despite the fact that the classic NRM achieves rapid convergence, one of the most prevalent problems is that the closest root is exceeded [15]. The NRM and the BM have been combined to form the new method [16]. The newly proposed strategy approaches the root in a progressive manner, eliminating the possibility of several iterations in the process. After a few iterations, the root is found using this new method, and the convergence process leads to the root that is the closest to the starting point.

1.2 Development of Modified Iterative Method

The MIM is different from the previous version of the NRM [17]. There is some modification done in the required method.

Let suppose the Non-linear Eq.

$$f(x) = 0 \quad (1)$$

Where α is the root of the above Eq., the function has a well-defined derivative and it is a continuous function [18].

By using Taylor's expansion

$$f(x) = f(x_n) + (x - x_n)f^{(1)}(x_n) + \frac{(x-x_n)^2}{2!}f^{(2)}(x_n) + \cdots, \quad (2)$$

Where,

x_n is the nth approximation.

Consider in Eq. (2) that α be the root

$$f(\alpha) = f(x_n) + (\alpha - x_n)f^{(1)}(x_n) + \frac{(\alpha-x_n)^2}{2!}f^{(2)}(x_n) + \cdots \quad (3)$$

Apply Eq. (1) here then we can get,

$$0 = f(x_n) + (\alpha - x_n)f^{(1)}(x_n) + \frac{(\alpha-x_n)^2}{2!}f^{(2)}(x_n) + \cdots \quad (4)$$

Now consider the linear form in Eq. (4), we can get the value of α

$$0 = f(x_n) + (\alpha - x_n)f^{(1)}(x_n)$$

$$\alpha = x_n - \frac{f(x_n)}{f^{(1)}(x_n)} \quad (5)$$

We also find similar results through the NRM. The initial guess of the root is x_0. If the first point x_0 is closed enough to α then the formerly the iteration (1) will converge to α. This method has the local convergence property [19].

Let suppose that the value of α can be represented by y_n , so our above function become like this,

$$y_n = x_n - \frac{f(x_n)}{f^{(1)}(x_n)} \quad (6)$$

The predetermined modification and calculation of $f^{(1)}(x_n)$ can be clearly explained by the basic principle of derivative [20], which is as follows.

$$f^{(1)}(x_n) \approx \frac{f(y_n)-f(x_n)}{y_n - x_n} \quad (7)$$

By putting Eq. (7) in Eq. (4), then we get

$$0 = f(x_n) + (\alpha - x_n)\left(\frac{f(y_n) - f(x_n)}{y_n - x_n}\right) + \frac{(\alpha - x_n)^2}{2!} f^{(2)}(x_n) \ldots,$$

$$-(\alpha - x_n) = \frac{y_n - x_n}{f(y_n) - f(x_n)}\left(f(x_n) + \frac{(\alpha - x_n)^2}{2!} f^{(2)}(x_n) \ldots,\right)$$

$$x_n - \frac{y_n - x_n}{f(y_n) - f(x_n)}\left(f(x_n) + \frac{(\alpha - x_n)^2}{2!} f^{(2)}(x_n) \ldots,\right) = \alpha$$

$$\alpha = x_n - \frac{y_n - x_n}{f(y_n)-f(x_n)}\left(f(x_n) + \frac{(\alpha-x_n)^2}{2!} f^{(2)}(x_n) \ldots,\right) \quad (8)$$

By putting Eq. (6) in Eq. (8), then we get

$$\alpha = x_n - \frac{\left(x_n - \frac{f(x_n)}{f^{(1)}(x_n)}\right) - x_n}{f(y_n) - f(x_n)}\left(f(x_n) + \frac{(\alpha - x_n)^2}{2!} f^{(2)}(x_n) \ldots,\right)$$

$$\alpha = x_n - \frac{\left(\frac{x_n f^{(1)}(x_n) - f(x_n) - x_n f^{(1)}(x_n)}{f^{(1)}(x_n)}\right)}{f(y_n) - f(x_n)}\left(f(x_n) + \frac{(\alpha - x_n)^2}{2!} f^{(2)}(x_n) \ldots,\right)$$

$$\alpha = x_n - \frac{\left(\frac{x_n f'(x_n) - f(x_n) - x_n f'(x_n)}{f'(x_n)}\right)}{f(y_n) - f(x_n)}\left(f(x_n) + \frac{(\alpha - x_n)^2}{2!} f''(x_n) \ldots,\right)$$

$$\alpha = x_n - \frac{\left(\frac{-f(x_n)}{f^{(1)}(x_n)}\right)}{f(y_n) - f(x_n)}\left(f(x_n) + \frac{(\alpha - x_n)^2}{2!} f^{(2)}(x_n) \ldots,\right)$$

$$\alpha = x_n - \frac{-f(x_n)}{[f^{(1)}(x_n)\{f(y_n) - f(x_n)\}]}\left(f(x_n) + \frac{(\alpha - x_n)^2}{2!} f^{(2)}(x_n) \ldots,\right)$$

$$\alpha = x_n + \frac{[f(x_n)]^2}{[f^{(1)}(x_n)\{f(y_n)-f(x_n)\}]} + \frac{(\alpha-x_n)^2}{2!} \frac{f(x_n)}{[f^{(1)}(x_n)\{f(y_n)-f(x_n)\}]} f^{(2)}(x_n) \ldots \quad (9)$$

When the value of α is very close to the x_n then neglect the square of the difference [21].

Now obtain the value of the second derivative by Eq. (6)

$$0 = f(x_n) + (\alpha - x_n)f^{(1)}(x_n) + \frac{(\alpha - x_n)^2}{2!} f^{(2)}(x_n)$$

$$\frac{-(\alpha - x_n)^2}{2!} f''(x_n) = f(x_n) + (\alpha - x_n)f'(x_n)$$

$$f^{(2)}(x_n) = \frac{-2}{(\alpha - x_n)^2}\left\{f(x_n) + (\alpha - x_n)f^{(1)}(x_n)\right\}$$

$$f^{(2)}(x_n) = \frac{-2f(x_n)}{(\alpha-x_n)^2} + \left\{\frac{-2}{(\alpha-x_n)^2}(\alpha - x_n)f^{(1)}(x_n)\right\} \quad (10)$$

For initial approximation, let suppose that in Eq. (10), α can be replaced by β.

Therefore,

$$f^{(2)}(x_n) = \frac{-2f(x_n)}{(\alpha-x_n)^2} + \left\{\frac{-2}{(\alpha-x_n)^2}(\beta - x_n)f^{(1)}(x_n)\right\} \quad (11)$$

By putting Eq. (11) in Eq. (9) we found that the β has the following relationships,

$$\beta = x_n + \frac{1}{f^{(1)}(x_n)}\left[\left\{\frac{f(y_n)}{f(x_n)}\right\}^2 [f(y_n) - f(x_n)] - f(x_n)\right] \quad (12)$$

Thus, Eq. (9) can be represented in Eq. (11) and (12) form,

$$\alpha = x_n + \frac{1}{f^{(1)}(x_n)}\left[\frac{[f(x_n)]^2}{[f(y_n)-f(x_n)]} - \frac{[f(y_n)]^2}{f(x_n)} + \quad \ldots\right] \quad (13)$$

Therefore, the preceding formula can be applied for the purpose of approximately determining the root of the Eq. (1). After reducing the number of terms to just two, the variable x_{n+1} will stand in for the iteration constant in this Eq.. [22]. Thus,

$$x_{n+1} = x_n + \frac{1}{[f^{(1)}(x_n)]}\left[\frac{\{f(x_n)\}^2}{[\{f(y_n)-f(x_n)\}]} - \frac{\{f(y_n)\}^2}{f(x_n)}\right] \quad (14)$$

Hence, Eq. (14) can be utilized for finding the root of Eq. (1) which is up to the required level of accuracy.

1.3 Rate of Convergence

For the MIM, the following convergence theorem demonstrates the theoretical significance of the choice of the initial point [23].

Let suppose that $n > 0$ then the term is given in Eq. (14)

$$x_{n+1} = x_n + \frac{1}{[f^{(1)}(x_n)]}\left[\frac{\{f(x_n)\}^2}{[\{f(y_n) - f(x_n)\}]} - \frac{\{f(y_n)\}^2}{f(x_n)}\right]$$

Converges to the simple zero of the defined function $f(x) = 0$.

To prove the above statement, we consider that $n \to \infty : f(x_n) \to 0$. According to the above Eq. which is iterative strategy that is why $n \to \infty$ makes $x_{n+1} \approx x_n$ [24]. Thus, apply these conditions on Eq. (14), so it converts the Eq. in this form which is given below,

$$\{f(x_n)\}^3 \to [\{f(y_n)\}^2\{f(y_n) - f(x_n)\}] \tag{15}$$

For the larger value of n, to prove the above result the NRM confirms that $f(y_n) \to f(x_n)$.

Consider one another condition that the function $f: 1 \subset R \to R$ can be differentiated and it has modest root α in the open interval. Hence the procedure is given in the Eq. (14) has 4th order convergence of the given root where $n > 0$ [25].

Now to prove this statement we consider that, $Z_n = x_n - \alpha$. So by using Taylor's expansion. We can write,

$$f(\alpha) = f(x_n) - f^{(1)}(x_n)Z_n + \frac{1}{2}f^{(2)}(x_n)Z_n^2 - \frac{1}{6}f^{(3)}(x_n)Z_n^3 + O(Z_n^4) \tag{16}$$

Hence, α is the real root of the function defined in Eq. (1) so that it is equal to zero.

$$0 = f(x_n) - f^{f^{(1)}}(x_n)Z_n + \frac{1}{2}f^{(2)}(x_n)Z_n^2 - \frac{1}{6}f^{(3)}(x_n)Z_n^3 + O(Z_n^4) \tag{17}$$

$$f^{(1)}(x_n)Z_n - \frac{1}{2}f^{(2)}(x_n)Z_n^2 + \frac{1}{6}f^{(3)}(x_n)Z_n^3 + O(Z_n^4) = f(x_n)$$

$$f(x_n) = f^{(1)}(x_n)Z_n + \frac{1}{2}f^{(2)}(x_n)Z_n^2 - \frac{1}{6}f^{(3)}(x_n)Z_n^3 + O(Z_n^4) \tag{18}$$

It can also be written as,

$$\eta_n = \frac{f(x_n)}{f^{(1)}(x_n)} = Z_n - \frac{1}{2}\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}Z_n^2 + \frac{1}{6}\frac{f^{(3)}(x_n)}{f^{(1)}(x_n)}Z_n^3 + O(Z_n^4) \tag{19}$$

Taylor's expansion can also be written as,

$$f(y_n) = f(x_n) - f^{(1)}(x_n)\eta_n + \frac{1}{2}f^{(2)}(x_n)\eta_n^2 - \frac{1}{6}f^{(3)}(x_n)\eta_n^3 + O(\eta_n^4) \tag{20}$$

Hence,

$$f(y_n) - f(x_n) = -f^{(1)}(x_n)\left[\eta_n - \frac{1}{2}\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}\eta_n^2 + \frac{1}{6}\frac{f^{(3)}(x_n)}{f^{(1)}(x_n)}\eta_n^3\right] + O(\eta_n^4) \tag{21}$$

By putting the value of Eq. (19) in the above Eq. so we get,

$$f(y_n) - f(x_n) = -f^{(1)}(x_n)\left[Z_n - \frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}Z_n^2 + \left\{\frac{1}{3}\frac{f^{(3)}(x_n)}{f^{(1)}(x_n)} + \frac{1}{2}\left(\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}\right)^2\right\}Z_n^3\right] + O(Z_n^4) \tag{22}$$

From Eq. (19)

$$\eta_n = \frac{f(x_n)}{f^{(1)}(x_n)}$$

We can evaluate from it,

$$\eta_n f^{(1)}(x_n) = f(x_n)$$

$$f(x_n) = \eta_n f^{(1)}(x_n) \tag{23}$$

From Eq. (22), we can get

$$f(y_n) - f(x_n) = -f^{(1)}(x_n)Z_n\left[1 - \frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}Z_n + \left\{\frac{1}{3}\frac{f^{(3)}(x_n)}{f^{(1)}(x_n)} + \frac{1}{2}\left(\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}\right)^2\right\}Z_n^2\right] + O(Z_n^4) \tag{24}$$

By putting Eq. (23), (24) and (20) in Eq. (14). Also, use $Z_n = x_n - \alpha$ then we get

$$Z_{n+1} = Z_n + \frac{1}{f^{(1)}(x_n)}\left[\frac{[\eta_n f^{(1)}(x_n)]^2}{-f^{(1)}(x_n)Z_n\left[1 - \frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}Z_n + \left\{\frac{1}{3}\frac{f^{(3)}(x_n)}{f^{(1)}(x_n)} + \frac{1}{2}\left(\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}\right)^2\right\}Z_n^2\right] + O(Z_n^4)} - \frac{\left[f(x_n) - f^{(1)}(x_n)\eta_n + \frac{1}{2}f^{(2)}(x_n)\eta_n^2 - \frac{1}{6}f^{(3)}(x_n)\eta_n^3 + O(\eta_n^4)\right]^2}{\eta_n f^{(1)}(x_n)}\right]$$

$$Z_{n+1} = Z_n + \frac{1}{f'(x_n)}\left[\frac{[\eta_n f'(x_n)]^2}{-f'(x_n)Z_n\left[1-\frac{f''(x_n)}{f'(x_n)}Z_n + \left\{\frac{1}{3}\frac{f'''(x_n)}{f'(x_n)}+\frac{1}{2}\left(\frac{f''(x_n)}{f'(x_n)}\right)^2\right\}Z_n^2\right]+O(Z_n^4)} - \frac{\left[\eta_n f'(x_n) - f'(x_n)\eta_n + \frac{1}{2}f''(x_n)\eta_n^2 - \frac{1}{6}f'''(x_n)\eta_n^3 + O(\eta_n^4)\right]^2}{\eta_n f'(x_n)}\right]$$

$$Z_{n+1} = Z_n - \left[\frac{[\eta_n]^2}{Z_n\left[1-\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}Z_n + \left\{\frac{1}{3}\frac{f^{(3)}(x_n)}{f^{(1)}(x_n)}+\frac{1}{2}\left(\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}\right)^2\right\}Z_n^2\right]+O(Z_n^4)} - \frac{\left[\frac{1}{2}f^{(2)}(x_n)\eta_n^2 - \frac{1}{6}f^{(3)}(x_n)\eta_n^3 + O(\eta_n^4)\right]^2}{\eta_n\{f^{(1)}(x_n)\}^2}\right]$$

$$Z_{n+1} = Z_n - \left[\frac{\eta_n^2}{Z_n\left[1-\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}Z_n + \left\{\frac{1}{3}\frac{f^{(3)}(x_n)}{f^{(1)}(x_n)}+\frac{1}{2}\left(\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}\right)^2\right\}Z_n^2\right]} - \frac{\left[\frac{1}{2}f^{2}(x_n)\eta_n^2 - \frac{1}{6}f^{(3)}(x_n)\eta_n^3 + O(\eta_n^4)\right]^2}{\eta_n\{f^{(1)}(x_n)\}^2} + O(Z_n^4)\right]$$

$$Z_{n+1} = Z_n - \left[\frac{\eta_n^2}{Z_n\left[1-\frac{f^{(2)}(x_n)}{f'(x_n)}Z_n + \left\{\frac{1}{3}\frac{f^{(3)}(x_n)}{f^{(1)}(x_n)}+\frac{1}{2}\left(\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}\right)^2\right\}Z_n^2\right]} - \frac{\eta_n^2\left[\frac{1}{2}f^{(2)}(x_n)\eta_n - \frac{1}{6}f^{(3)}(x_n)\eta_n^2 + O(\eta_n^3)\right]^2}{\eta_n\{f^{(1)}(x_n)\}^2} + O(Z_n^4)\right]$$

$$Z_{n+1} = Z_n - \left[\frac{\eta_n^2}{Z_n\left[1-\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}Z_n + \left\{\frac{1}{3}\frac{f^{(3)}(x_n)}{f^{(1)}(x_n)}+\frac{1}{2}\left(\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}\right)^2\right\}Z_n^2\right]} - \frac{\eta_n\left[\frac{1}{2}f^{(2)}(x_n)\eta_n - \frac{1}{6}f^{(3)}(x_n)\eta_n^2 + O(\eta_n^3)\right]^2}{\{f^{(1)}(x_n)\}^2} + O(Z_n^4)\right]$$

(25)

Eq (25) can be more simplified by binomial theorem and by using Eq. (19)

$$Z_{n+1} = Z_n - \left[Z_n - \frac{f^{(2)}(x_n)}{f'(x_n)}Z_n^2 + \frac{1}{3}\frac{f^{(3)}(x_n)}{f^{(1)'''}\quad{}_{pkikhand\ management}(x_n)}Z_n^3 + \frac{1}{4}\left\{\left(\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}\right)^2\right\}Z_n^3 + O(Z_n^4)\right]\left[1-\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}Z_n + \frac{1}{3}\frac{f^{(3)}(x_n)}{f^{(1)}(x_n)}Z_n^2 + \frac{1}{2}\left\{\left(\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}\right)^2\right\}Z_n^2 + O(Z_n^3)\right] - \frac{1}{4}\left\{\left(\frac{f^{(2)}(x_n)}{f^{(1)}(x_n)}\right)^2\right\}Z_n^3 + O(Z_n^4)$$

After solving the above Eq., we can find

$$Z_{n+1} = O(Z_n^4) \quad (26)$$

This sets up the 4th order convergence of the iterative strategy characterized by Eq. (14). Subsequently, a hypothesis is demonstrated [26].

3. Numerical Tests and Results

A small number of random Non-linear Eq.s, such as trigonometric, exponential, logarithmic, and cubic polynomial Eq.s, have been successfully solved [27]. Numerical experiments have been conceded in the Matlab environment. A variety of alternative approaches, such as the RFM, the NRM, and The Proposed Method-Modified Iterative Solver, have been used to find the roots, the number of iterations, the residuals, and the computational time for various functions [28]. In order to obtain numerical results, all four functions must be tested using the conventional test functions mentioned below, one after the other.

P1: $f(x) = \frac{Sin(x)+Cos(x)}{Cos(x)}$

P2: $f(x) = e^{2x} + 2x - 3$

P3: $f(x) = x^2 - log\ (x) + 5$

P4: $f(x) = x^3 + 2x^2 + x - 1$

Matlab was used to implement the answers to each of the four problems. The findings of the RFM, the NRM, and the Modified-iterative solver are presented in Tables 1, 2, 3, and 4. The tables below display the root, residuals, number of iterations, and computing time.

Table 1

Result for the function f(x) = (sin(x) + cos(x))/(cos(x))

P1 $f(x) = \frac{sin(x)+cos(x)}{cos(x)}$ Root = -0.78539	Residual $f(x^*)$	Solve Time (in Sec.)	No. of Iterations
RF	0.260158	0.016749	7
NR	0.284735	0.371802	4
MIM	12.49493	0.175360	2

Table 2

Result for the function for f(x) = e^2x+2x-3

P1 $f(x) = e^{2x} + 2x - 3$ Root = 0.396030	Residual $f(x^*)$	Solve Time (in Sec.)	No. of Iterations
RF	0.396518	0.003304	13
NR	0.607595	0.223741	4
MIM	0.018682	0.191202	2

Table 3

Result for the function for f(x) = x^2-log(x) +5

P1 $f(x) = x^2 - log\ (x) + 5$ Root = 2.067306	Residual $f(x^*)$	Solve Time (in Sec.)	No. of Iterations
RF	0.080793	0.001693	7
NR	0.003551	0.241817	5
MIM	9.882621	0.195330	2

Table 4

Result for the function f(x) =(x^3+2x) ^2+x-1

P1 $f(x) = x^3 + 2x^2 + x - 1$ Root = 0.465571	Residual $f(x^*)$	Solve Time (in Sec.)	No. of Iterations
RF	1.101277	0.002629	9
NR	0.393042	0.209061	4
MIM	0.000030	0.220651	4

RFM: Regula-Falsi Method

NRM: Newton-Raphson Method

MIM: Modified-Iterative Method

By comparing the number of iterations required for the function to converge in the RFM and the NRM, we can observe that the number of iterations is increasing in both techniques. In such circumstances, the Matlab program loop becomes eternally confined by a limit on the number of iterations that can be performed [29]. Despite the fact that we can see from the above table that the contribution of each line to the stability of the function may not be the same [30], we can also see that the contribution of each line to the stability of the function may not be the same [31]. When the function of a line is changed, the stability of the function is more dependent on the lines that indicate a considerable increase in the number of iterations that we need, when the function is changed [32].

The outcomes presented in Tables 1, 2, 3, and 4 illustrate that as the number of intervals increases, the needed number of iterations reduces. Consequently, the MIM takes fewer iterations than the RFM and NRM, resulting in a lower computing time [27].

The RFM, the NRM, and The Modified Iterative Solver have slightly different computation times than the NRM and The Modified Iterative Solver. It is clear that the RFM takes less time to compute than the MIM when the computational time is compared between the two methods. We can see that The Modified Iterative technique takes less time when we compare computing time between the NRM and The MIM, as well. We can also observe that there is a significant difference in residuals between the RFM and the MIM if we compare the two methods. The RFM has a slight error while The MIM has a significant error. If we look at the residuals of both the NRM and The MIM, we can see that there is only a tiny difference between the two methods.

The computational time, on the other hand, increased significantly as the gap expanded. Comparing the RFM with the NRM and The MIM requires much more processing time. The substantial increase in needed processing time is due to the partitioning of the interval and the calculation of $f(a)\, f(b) < 0$ in each interval [33].

The corresponding graphs of the function from Tables 1, 2, 3, and 4 are depicted in pictorial form given below.

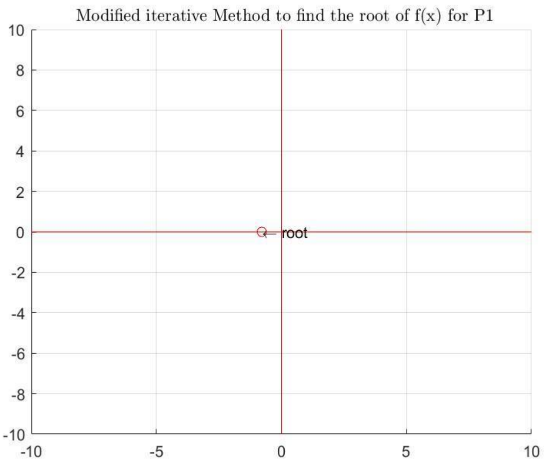

Fig. 1. MIM of P1

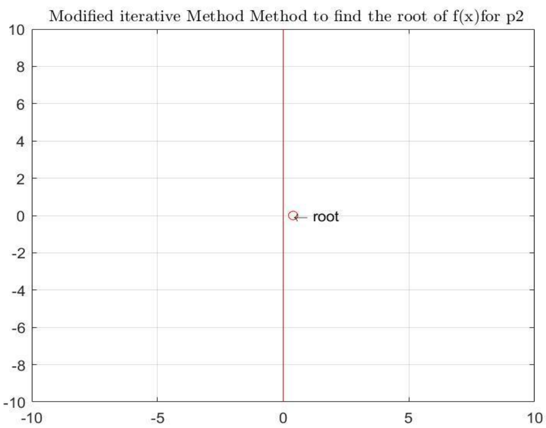

Fig. 2. MIM of P2

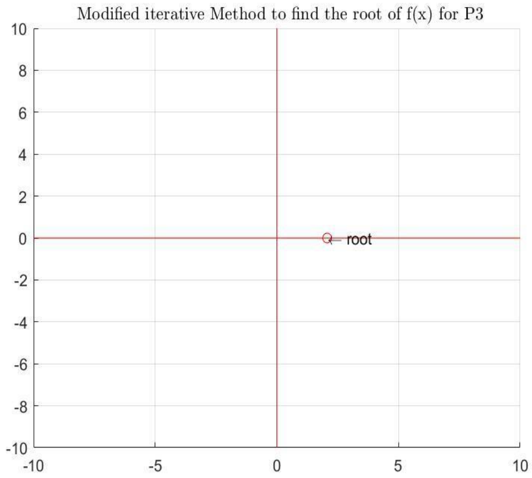

Fig. 3. MIM of P3

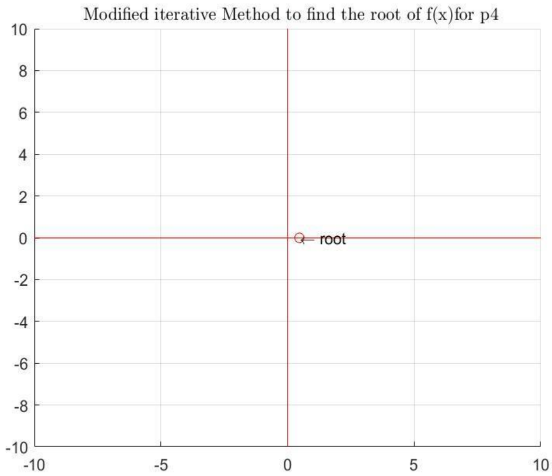

Fig. 4. MIM of P4

4. Discussion

The Matlab was used to implement the answers to each of the four problems. The findings for the RFM, the NRM, and the Modified-iterative solver are shown in Tables 1, 2, 3, and 4. The above tables display the root, residuals, number of iterations, and computing time.

The tolerance threshold is 1e-05, and the initial estimation is 0.1, which is a reasonable estimate. If you want to avoid any complications, set the number of iterations to 20 iterations before you start the iteration error process. The residual fix is set to 1e-05, which indicates that if the residual value is less than this value, the iteration process will end. The number of decimal places represents the root tolerance for the current iteration of the algorithm as well as previous iterations.[34], [35].

For initial assumptions near the critical point, the resultant solution was more than a root away from the critical point [36]. With each passing second, the overshooting grew as the initial guesses got closer to the critical point. The overshooting dropped as the initial predictions moved further from the crucial point, and the overshooting was reduced until it reached the closest root [37]. As a result of the stopping criteria employed in the calculations, the approach could converge to the root most closely associated with the region [38].

When using a numerical approach that meets certain conditions, the capacity to converge and the speed with which it converges are the most important steps to consider [39]. Previously, we discussed how the BM is a suitable approach for achieving guaranteed convergence [40]–[42]. It is possible to obtain the number of iterations requisite for a certain level of accuracy. NRM is somewhat similar, however, with a few minor differences [43]. In order to complete each step, a derivative must be calculated. At this point, we are presenting an iterative solver for Non-linear Eq.s that is method-modified and would achieve better convergence with more than previous techniques in terms of computing time [44].

Tables 1, 2, 3, and 4 depict the results of the RFM and the NRM, respectively. The first column indicates the residual, the second column reflects the length of processing time, and the third column represents the number of iterations. The residue is depicted in the first column.

When the initial guess was in the region, as shown in Tables 1, 2, 3, and 4, the approach converges to the root that is closest to the starting point. For initial assumptions that were near to the critical point, the resultant solution was more than a root away from the critical point [28]. The overshooting became more pronounced as the first estimations got closer to the critical moment. With each step forward in distance from the critical point, the overshooting decreased until the initial guess reached the root that was the closest to that point [31]. As a result of the stopping criteria employed in the calculations, the approach converged to the root most closely associated with the region.

Based on the results of the RFM, the NRM, and the Modified Iterative solver in Tables 1, 2, 3, and 4, it can be concluded that the computational time varies for all three methods, namely the RFM, the NRM, and the Modified Iterative solver. Comparing the computing times of the RFM with the MIM reveals that the Regula-Falsi method requires less time to calculate than the MIM. Comparing the computational times of the NRM with the MIM reveals that the MIM requires less time.

We can also observe a significant difference in residuals between the RFM and the MIM if we compare the two methods. The RFM has a slight error while the MIM has a significant error. If we look at the residuals of both the NRM and the MIM, we can see that there is only a tiny difference between the two methods.

Iterative methods require a significant amount of processing time in order to converge on the closest root, which is a major factor in determining their overall performance [45]. This fact provides valuable information into the performance of each strategy in question.

5. Conclusion

The RFM, NRM, and SM are based on the same underlying concept, with only minor implementation differences. Convergence of other two is sensitive with respect to the initial guess. The RFM and SM do not need the computation of derivatives at any point of the process, but the NRM requires the computation of a derivative at each stage. Although the RFM and the SM are more computationally costly than the NRM, they permit quicker convergence.

The proposed MIM approaches to root in fewer iterations, which is validated by numerical results presented in relevant section. Also, the MIM converges to desired accuracy in a short extent of time than competitors. The trip of this study was divided into three stages: the first stage involved the interpretation of the RFM, the second stage involved the application of the NRM, and the last stage involved the application of modifications to the iterative process. We have developed a mechanism to accelerate the convergence of the NRM, which can result in a significant increase in the order of convergence. Applicability and performance of MIM on different types of nonlinear Eq.s have been tested. Using four distinct functions, we compared our novel method, the Modified Iterative solver, with the RFM and NRM, and analysed the results. The examples indicate that The Modified Iterative Solver is a moderately quicker approach that requires less iteration steps and fewer functional computations than the conventional iterative solver.

Following this investigation, we can conclude that MIM is better choice as compared to classical methods in terms of iterations and computational time.

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7. References

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