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Mathematical modeling and optimization of wastewater stabilization ponds using nonlinear programming

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ABSTRACT

This research work proposes a mathematical optimization model for constructing two ponds (1. Facultative pond and 2. Maturation pond) provided in series. The model uses concrete volume as its objective minimization function. There were two decision variables; the first was the detention time (D_T) and the second in the list was the number of provided baffle walls (N_{BW}) in both ponds. The constraint parameters include fecal coliforms and biochemical oxygen demand (BOD_5). The model was applied with the help of an Add-Ins of MS Office: Excel solver. The generalized reduced gradient algorithm was utilized in the solver (GRG). Before applying the mathematical optimization model, ponds were designed with the conventional method and then using optimized values of the variables. A comparison of the findings reveals that a 13.79 percent reduction in the D_T, an 11.55 percent reduction in the area, and a 7.19 percent reduction in the volume of concrete occurred. The reduction values mentioned above are significant since these systems' fundamental drawback is the area's requirement. In addition, sensitivity analyses of the objective function and the removal of pollutants are also provided. The model described above is sensitive to variations in the parameters. Both analyses demonstrated that the effluent characteristics comply with the class-B irrigation standards in Turkey. It is advised to do more optimization studies for WSPs with the help of other algorithms and tools available in the literature for distinct wastewater treatment plants.

1. Introduction

Wastewater generated from any source is a significant public and environmental health concern [1]. The environment will be polluted if the wastewater is discharged without adequate treatment [2]. Moreover, the major hurdle in the sustainable development of any human society is the water supply crisis and pollution control [3]. Since the 1990s, water quality has worsened in almost all the lakes and rivers in Asia, Latin America, and Africa [4]. Over the next decade, this deterioration is projected to intensify further [4]. The scenario above highlights the need for more wastewater treatment plants worldwide, particularly in the continents mentioned above.

Wastewater stabilization ponds (WSPs) are provided with three primary goals: a) the removal of fecal coliforms, b) the elimination of biochemical oxygen demand (BOD), and c) the removal of nutrients: nitrogen and phosphorus. WSPs are recommended in countries with tropical climates due to the environmental factors that enhance the removal efficiency of contaminants from wastewater that passes through them. However, they are also provided in areas having a cold climate [5-6]. The main disadvantage of their provision is the requirement of the area [6]. So this highlights the need to explore various techniques that can help reduce the area requirement for WSPs. The purpose of this study was to address this lacuna in the existing studies.

WSPs are constructed using various materials such as soil, gravel, steel, and concrete [8-9]. They have several types: aerobic, aerobic maturation, anaerobic and facultative ponds. They mainly have three diverse flow types: complete mix, dispersed, and plug flow [10]. In the present study, it was assumed that concrete would be used to construct WSPs. The ponds were designed for a village representing the typical population and meteorological conditions near Antalya city of Turkey. As mentioned above, the main hurdle in their provision is the area requirement that ultimately requires more concrete volume. So, the reduction in the overall area can reduce the needful volume of concrete.

Goodarzi et al. [11] claim that baffle walls (BWs) in pond systems can improve flow conditions, reduce dead sections in the ponds, and enhance pollution removal efficiency. Several studies have been conducted to check their effectiveness at various numbers and lengths. Li et al. [12] have also examined their impact at various numbers, lengths, and spacing between them. Additionally, he has addressed the research of various scholars who have studied the influence of BWs on WSPs. Another observation by Goodarzi et al. [11] is that the hydraulic and treatment efficiency of WSPs also improves with the addition of BWs. Therefore, their presence increases the efficiency of wastewater treatment by providing plug flow. Martinez et al. [5] investigated the impact of BWs to minimize the overall area required. He also listed other authors who have worked on reducing the area using various techniques. The study concluded that the addition of BWs reduces the land requirement.

On the other hand, it increases the construction cost [5]. The scenario explained above makes optimization studies using various alternatives necessary. In this study, the author used the Excel solver to optimize the size of facultative and maturation ponds by applying some design and effluent constraints. The system analyzes the mathematical optimization model using the generalized reduced gradient (GRG) algorithm [13]. The solver examines and modifies variables until constraints are satisfied [14]. The author recently published another article on optimizing concrete volume for facultative ponds using the excel solver and GRG algorithm [15].

The primary aim of this research was to minimize the volume of the concrete needed for the construction of WSPs. Following were the objectives: 1. Design WSPs using the traditional approach. 2. Write a mathematical model for the two ponds to be constructed in series. 3. Optimize the variables considering relevant constraints with the help of MS excel solver and GRG algorithm. 4. Design WSPs using the optimized values and compare the results.

2. Methodology

2.1 Acronyms and Abbreviations

MPN, Most probable number; $(N_{BW})_{FP}$, Number of baffle walls in facultative pond; $(N_{BW})_{MP}$, Number of baffle walls in maturation pond; L_{BW}, Baffle walls' length; BWs, Baffle walls; WSPs, Waste/Wastewater Stabilization Ponds; FPs, Facultative Pond; MP, Maturation Pond; $(D_T)_{FP}$, Hydraulic detention time of facultative pond; $(D_T)_{MP}$, Hydraulic detention time of maturation pond; $(Q_i)_{FP}$, Inflow of the facultative ponds (m^3/d); $(Q_i)_{MP}$, Inflow of the maturation ponds (m^3/d); $Q_e)_{FP}$, outflow of the facultative ponds after evaporation correction (m^3/d); $Q_e)_{MP}$, outflow (m^3/d) of the maturation pond after evaporation correction; $(BOD_5)_i$, Biochemical oxygen demand on 5th day and at influent (mg/l); $(BOD_5)_e$, Biochemical oxygen demand on 5th day and after applying the evaporation correction (mg/l); T_{avg}, Average air temperature calculated from last ten years metrological data of the study site (°C); V_p, Calculated volume of the ponds (m^3); d_{FP}, Depth of the facultative pond (m); d_{MP}, Depth of the maturation pond (m); t; The assumed concrete thickness of both slab and walls; A_P, Area of the ponds in total (m^2); K_b, Bacterial decay constant (d^{-1}); N_i (MPN/100 mL), Influent Fecal coliform; N_f/N_o; Effluent fecal coliform (MPN/100 mL); N_e; Effluent fecal coliform (MPN/100 mL); X, Length to width ratio; W_{FP}, Facultative pond's width (m); L_{FP}, Facultative pond's length (m); W_{MP}, Maturation pond's width (m); L_{MP}, Maturation pond's length (m); A_{FP}, Facultative pond's area (m^2); A_{MP}, Maturation pond's area (m^2); V_{conc}, Concrete volume (m^3); d_f, Factor for the dispersion in both ponds; a, A constant without dimensions; λ_v, Volumetric organic load rate (g/m^3/d); λ_s, Organic surface loading rate (kg/ha.d).

2.2 Design Procedure

Yanez's approach for dispersed flow conditions was used to design both ponds. In their research paper, Martinez et al. [5] have written the whole design process for these two ponds. The process for the design of WSPs included in this research work is similar to their work with some changes, as mentioned below.

2.2.1 Anaerobic Pond

a. Volumetric organiv load $\left(\frac{g.\ BOD_5}{m^3.\ d}\right) = \lambda_v = 20 \times T_{avg} - 100$ (1)

b. BOD_5 removal efficiency (%) $= 2 \times T_{avg} + 20$ (2)

2.2.2 Facultative and Maturation ponds

a. Using the following equation, the extreme organic surface loading rate is computed.

$$\lambda s \left(\frac{kg}{ha.d}\right) = 350 \text{ x } \left(1.107 - 0.002 \text{ x } T_{avg}\right)^{T_{avg}-25} \quad (3)$$

Equation 3 incorporates factors of safety to provide a design equation that is globally applicable [16].

b. Additionally, the coefficient of bacterial reduction was calculated as mentioned below. First, $(K_b)_{20}$ was computed using the depth of the FP (1.5 m) and MPs (1 m). Then, $(K_b)_{T_{avg}}$ was determined based on the coldest month's average temperature for the past ten years.

$$(k_b)_{T_{avg}} = (k_b)_{20} \times \theta^{\,T_{avg}-25} \quad (4)$$

Where: $(k_b)_{20}$ was calculated using this equation, $0.542 \text{ x } H^{-1.259}$. The θ value was taken constant; Marais 1974 suggested 1.19. However, later on Yanez 1993 revealed that the value was miscalculated and should be taken equivalent to 1.07 [17].

2.3 Mathematical optimization model

The Microsoft Excel solver followed the GRG algorithm to optimize the amount of concrete needed to construct both ponds. The concrete volume (V_{conc}) was taken as the objective function that was expressed in the form of concrete volume needed for the BWs, the parameter walls and the base slab. Below mentioned are the steps followed to develop the mathematical model for the optimization of concrete volume. The hypothesis was that the mathematical model will be developed with the help of the relationship between various variables such as detention time (D_T), the number of baffle walls (N_{BW}), and length of baffle walls (L_{BW}). The dimensions of the base slab, parameter walls, and BWs are given in terms of these in equations 13 and 14. The objective function of this optimization model was given by the equation 14.

$$\begin{aligned}
&\text{Minimize the total concrete volume for the ponds}\\
&(\text{FP} + \text{MP}) = Min.V_{conc} =\\
&\Big(\big(\text{Concrete volume for the floor slab of FP } \big((L_{FP} \times W_{FP}) \times t\big) +\\
&\text{Concrete volume for the side walls of FP } \big((2 \times L_{FP} \times d_{FP} + 2 \times W_{FP} \times d_{FP}) \times t\big) +\\
&\text{Concrete volume for BWs of FP } (L_{FP} \times d_{FP} \times N_{BW} \times \text{Percentage length of the BWs in FP}\,) +\\
&\big(\text{Concrete volume for the floor slab of MP } \big((L_{MP} \times W_{MP}) \times t\big) +\\
&\text{Concrete volume for the side walls of MP } \big((2 \times L_{MP} \times d_{MP} + 2 \times W_{MP} \times d_{MP}) \times t\big) +\\
&\text{Concrete volume for BWs of MP } (L_{MP} \times d_{MP} \times N_{BW} \times \text{Percentage length of the BWs in MP}\,)\Big) \quad (5)
\end{aligned}$$

The thicknesses of the walls and floor slabs of both ponds were deemed equivalent (t = 15 cm). To simplify, t was assumed to be constant, and the equation was altered, as shown below.

$$Min.V = \Bigg((L_{FP} \times W_{FP}) + (2 \times L_{FP} \times d_{FP} + 2 \times W_{FP} \times d_{FP}) + \left(\left(\frac{L_{BW}}{L_{FP}} \times 100\right) \times L_{FP} \times (N_{BW})_{FP} \times d_{FP}\right) + (L_{MP} \times W_{MP}) + (2 \times L_{MP} \times d_{MP} + 2 \times W_{MP} \times d_{MP}) + \left(\left(\frac{L_{BW}}{L_{MP}} \times 100\right) \times L_{MP} \times (N_{BW})_{MP} \times d_{MP}\right)\Bigg) \times \text{t} \quad (6)$$

Following steps were taken to convert the size of both ponds into design variables.

Average hydraulic detention time:

$$(D_T)_{FP} = \frac{V_{FP}}{(Q_i)_{FP}} \quad (7)$$

$$V_{FP} = A_{FP} \times d_{FP} \quad (8)$$

Length (L_{FP}) can be determined for FPs using the formulas shown below if the ratio between length and width is 3.

$$L_{FP} = 3 \times W_{FP} \quad (9)$$

The depth of both walls (parameter and baffle) was equivalent to the depth of pond and represented as (d_p) = 1.5 m for FP and 1 m for MP. For aesthetic purpose width of both ponds was considered equal and calculated as given below.

$$W_{FP} = W_{MP} = \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{3 \times d_p}} = \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}} \quad (10)$$

$$L_{FP} = 3 \text{ x } \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}} \quad (11)$$

$$L_{MP} = \frac{A_{MP}}{W_{MP}} \quad (12)$$

After deciding that the L_{BW} of both ponds was equal to 0.7, equation 6 was revised to the form shown below by replacing the pond dimensions with this value.

$$\text{Min. V} = \Bigg(\bigg(3 \times \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}} \times \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}}\bigg) + \Bigg(\bigg(2 \times \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}}\bigg) \times 1.5 + 3 \times \bigg(2 \times \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}}\bigg)\Bigg) \times 1.5 + 3 \times 1.5 \times (N_{BW})_{FP} \times 0.7 \times \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}} + \bigg(\frac{A_{MP}}{W_{MP}} \times \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}}\bigg) + \Bigg(\bigg(2 \times \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}}\bigg) \times 1 + \bigg(2 \times \frac{A_{MP}}{W_{MP}}\bigg)\Bigg) \times 1 + 1 \times (N_{BW})_{MP} \times 0.7 \times \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}}\Bigg) \times t \quad (13)$$

Following the principles for square root multiplication and multiplying the other components, equation 13 may be further reduced, as shown below.

$$\text{Min. V} = \Bigg(\bigg(3 \times \frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}\bigg) + \bigg(11 \times \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}}\bigg) + 3.15 \times (N_{BW})_{FP} \times \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}} + \bigg(\frac{A_{MP}}{W_{MP}} \times \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}}\bigg) + \Bigg(\bigg(2 \times \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}}\bigg) + \bigg(2 \times \frac{A_{MP}}{W_{MP}}\bigg)\Bigg) + 0.7 \times (N_{BW})_{MP} \times \sqrt{\frac{(D_T)_{FP} \times (Q_i)_{FP}}{4.5}}\Bigg) \times t \quad (14)$$

It is crucial to note here that the discharge taken for the design of ponds is not counted as decision variable. Instead, it is used to calculate the dimensions of the ponds. Following are the design and optimization constraints.

$(N_{BW})_{FP} \leq 10$,

$(N_{BW})_{MP} \leq 4$

N_{BW} (for both ponds) = Integer,

$30 \leq (D_T)_{FP} \leq 50$ days,

$18 \leq (D_T)_{MP} \leq 20$ days,

N_{BW}, D_T, and d_f (of both ponds) > 0.

$(BOD_5)_e \leq 30$ mg/l,

Fecal coliforms ≤ 200 MPN/100mL,

2.4 Application of the mathematical optimization model

The ponds were designed for a village area close to Antalya city. The flow rate (Qi) used was 214.8 m^3/day. According to the Turkish meteorological department in Trabzon, Turkey, the average temperature of the coldest month in the study area, calculated from last ten years data was 10.2 (℃) and the evaporation calculated was 5.3 mm/day. Fecal coliform concentration at influent was 10^7 MPN/100 mL, whereas BOD_5 in the influent was 340 mg/L. These are the average values for the wastewater produced from a domestic source [18].

To assess the appropriateness of the effluents, Turkey's Irrigation standards (Class-B) were taken into account. According to the regulations, effluent $(BOD_5)_e$ must be less than 30 mg/L, and the effluent concentration of fecal coliforms be required to be smaller than 200 MPN/100 mL. As was mentioned above, the model restrictions dictated a range of 4-10 BWs in the facultative pond. In addition, their length was taken as 70% of the overall length of each pond. The 70 % length was based on the recommendation of most optimization studies [19]. In addition, it was ensured that N_{BW}, D_T, and d_f are higher than zero and that BWs are integers. In the Turkish design guidelines for facultative ponds, the maximum and lowest D_T varied from 30 to 50 days for FP and 18 to 20 days for MP [20].

3. Results and Discussion

Table 1 displays the design inputs and outputs of the ponds using the traditional method. Total area, D_T, and V_{conc} were 10672.97 m^2, 69.81 days, and 1801. 69 m^3, respectively. There were 2 and 4 BWs provided, before applying the mathematical model, in FPs and MPs, respectively, when the desired effluent standards were achieved [20].

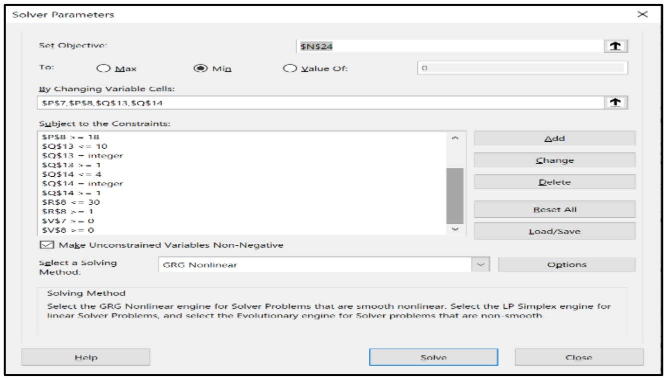
Table 1

Results with the Traditional Method

Input Data			
Q_i	N_i	$(BOD_5)_i$	T_{avg}
214.80	10000000	340	10.2
Facultative Pond		Maturation Pond	
D_T	BWs	D_T	BWs
49.81	2	20	4
Output			
Facultative Pond			
X	d_f	a	L
19	0.0516	1.65	146.28
W	Ne	Q_e	$(BOD_5)_e$
48.76	21003	176.99	46
Area	V_{conc}		
7133.07	1203.81		
Maturation Pond			
X	d_f	a	L
26	0.03752048	1.36	72.60
W	Ne	Q_e	$(BOD_5)_e$
48.76	200	158.23	12
Area	V_{conc}		
3539.89	597.88		
Total	Area	D_T	V_{conc}
	10672.97	69.81	1801.69

3.1 Mathematical model optimization

Table 2 demonstrates the design calculations of WSPs with the optimized values. Cells of the objective function, variables, imposed restrictions, and the method used for the analysis may be seen in the solver parameters window shown in Fig. 1. The solver results window shows that the solution has been found by fulfilling all of the desired standards, displayed in Fig. 2. Solver's answer report window and the goal function's initial and end values are shown in Fig. 3. Furthermore, the number of iterations, time taken for the analysis, constraints satisfaction, names of variables with their relevant cell code, and binding status are also shown in Fig. 3.


Solver Parameters

Set Objective: N24

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells:
P7,P8,Q13,Q14

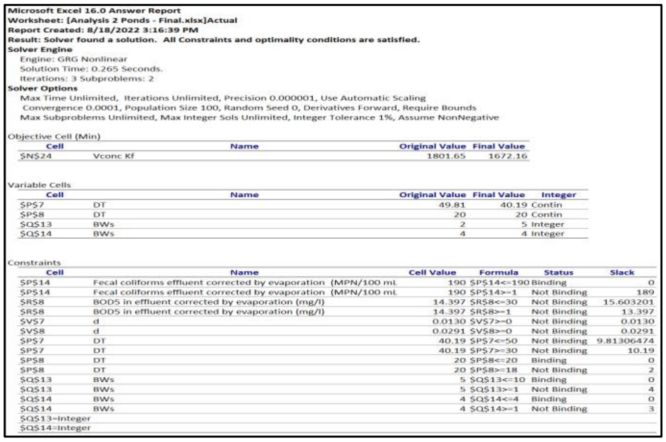
Subject to the Constraints:
P8 >= 18
Q13 <= 10
Q13 = integer
Q13 >= 1
Q14 <= 4
Q14 = integer
Q14 >= 1
R8 <= 30
R8 >= 1
V7 >= 0
V8 >= 0

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

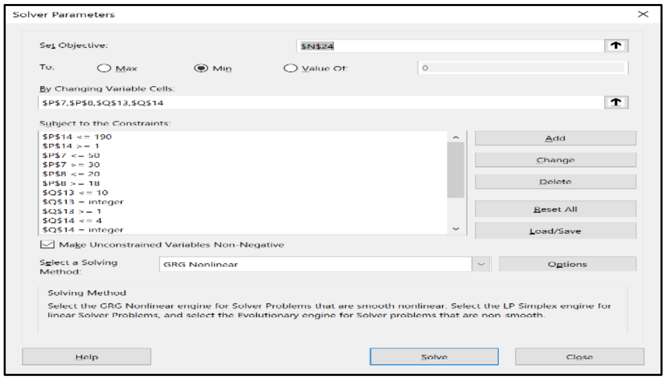
Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

(a) The set objective, variable cells, and constraints' first part


```
Microsoft Excel 16.0 Answer Report
Worksheet: [Analysis 2 Ponds - Final.xlsx]Actual
Report Created: 8/18/2022 3:16:39 PM
Result: Solver found a solution.  All Constraints and optimality conditions are satisfied.
Solver Engine
   Engine: GRG Nonlinear
   Solution Time: 0.265 Seconds.
   Iterations: 3 Subproblems: 2
Solver Options
   Max Time Unlimited,  Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
    Convergence 0.0001, Population Size 100, Random Seed 0, Derivatives Forward, Require Bounds
   Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)
     Cell                          Name                          Original Value  Final Value
  $N$24      Vconc Kf                                                   1801.65      1672.16

Variable Cells
     Cell                          Name                          Original Value  Final Value  Integer
  $P$7       DT                                                           49.81     40.19 Contin
  $P$8       DT                                                              20        20 Contin
  $Q$13      BWs                                                              2         5 Integer
  $Q$14      BWs                                                              4         4 Integer

Constraints
     Cell                          Name                          Cell Value  Formula       Status         Slack
  $P$14      Fecal coliforms effluent corrected by evaporation  (MPN/100 mL      190 $P$14<=190 Binding            0
  $P$14      Fecal coliforms effluent corrected by evaporation  (MPN/100 mL      190 $P$14>=1   Not Binding      189
  $R$8       BOD5 in effluent corrected by evaporation (mg/l)               14.397 $R$8<=30   Not Binding 15.603201
  $R$8       BOD5 in effluent corrected by evaporation (mg/l)               14.397 $R$8>=1    Not Binding    13.397
  $V$7       d                                                            0.0130 $V$7>=0    Not Binding    0.0130
  $V$8       d                                                            0.0291 $V$8>=0    Not Binding    0.0291
  $P$7       DT                                                            40.19 $P$7<=50   Not Binding 9.81306474
  $P$7       DT                                                            40.19 $P$7>=30   Not Binding     10.19
  $P$8       DT                                                               20 $P$8<=20   Binding            0
  $P$8       DT                                                               20 $P$8>=18   Not Binding        2
  $Q$13      BWs                                                               5 $Q$13<=10  Binding            0
  $Q$13      BWs                                                               5 $Q$13>=1   Not Binding        4
  $Q$14      BWs                                                               4 $Q$14<=4   Binding            0
  $Q$14      BWs                                                               4 $Q$14>=1   Not Binding        3
  $Q$13=Integer
  $Q$14=Integer
```

(b) The set objective, variable cells, and constraints' second part

Fig. 1. Answer report of Microsoft excel solver


Solver Parameters

Set Objective: N24

To: ○ Max ● Min ○ Value Of: 0

By Changing Variable Cells:
P7,P8,Q13,Q14

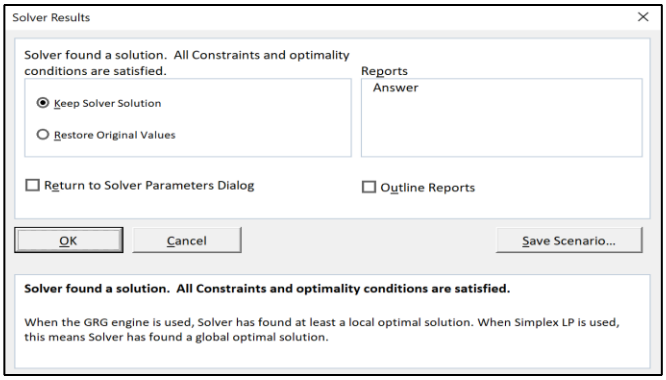
Subject to the Constraints:
P14 <= 190
P14 >= 1
P7 <= 50
P7 >= 30
P8 <= 20
P8 >= 18
Q13 <= 10
Q13 = integer
Q13 >= 1
Q14 <= 4
Q14 = integer

☑ Make Unconstrained Variables Non-Negative

Select a Solving Method: GRG Nonlinear

Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Fig. 2. Solver results showing constraints are satisfied


Solver Results

Solver found a solution. All Constraints and optimality conditions are satisfied.

● Keep Solver Solution
○ Restore Original Values

☐ Return to Solver Parameters Dialog

Reports: Answer

☐ Outline Reports

Solver found a solution. All Constraints and optimality conditions are satisfied.

When the GRG engine is used, Solver has found at least a local optimal solution. When Simplex LP is used, this means Solver has found a global optimal solution.

Fig. 3. Excel solver answer report window

The comparison of both designs shows that the application of the optimization model has reduced the volume of concrete by 129.54 m^3 (7.19 %), the area of the ponds by 1232.20 m^2 (11.55 %), and D_T by 9.63 days (13.79 %). This is a considerable number in light of the available economic resources. According to Martinez et al. [5], D_T directly affects the size of the pond and, thus, the area needed. This research's findings are consistent with the above mentioned author. There were 5 BWs in the facultative pond, as opposed to the two that were initially proposed. According to Goodarzi et al. [12], this helps the plug flow conditions. The findings of this research are consistent with the author's claim.

3.1.1 Facultative Pond

The design areas for the facultative pond by each technique are shown in Tables 1 and 2. The possible facultative pond dimensions in the mathematical model shows a reduction in the area, D_T, and V_{conc} by 1378.30 m^2 (19.32 %), 9.63 days (19.32 %), and 158.29 m^3 (13.15 %), respectively. The smaller area is also reflected in eliminating fecal coliforms. As previously said, the reduction achieved above is a consequence of the creative inclusion of five BWs instead of the two in the traditional method. It is possible to lower the cost of pond systems by optimizing the design, including practical limits and constantly considering the quality criteria for the effluent of these systems [20]. This investigation supports the claimed theory in the manuscript.

3.1.2 Maturation Pond

The findings of the design of the maturation pond are shown in Tables 1 and 2. Adopting the suggested mathematical model resulted in no change in N_{BW} and D_T. In contrast, a little increase in the area (146.10 m^2 or 4.13 %), and V_{conc} (28.75 m^3 or 4.81 %) is observed. Despite those mentioned above, overall reduction was observed, and the effluent fulfilled the class-B effluent quality standards for irrigation in Turkey [20]. According to Lian et al. [21], it is not sensible to arbitrarily increase the number of BWs. Instead, a cost-effectiveness economic study should be conducted. According to Rediske et al. [22], ideal findings are the ones in which decision-making judgment factors are achieved.

Table 2 reveals the design outcome using the mathematical optimization model and the deciding factors used in this research. The same table shows that effluent fecal coliforms have precisely reached the desired effluent standards. It is also apparent that the system selects a greater number of BWs compared to the traditional approach. In contrast, BODe is higher than that achieved in the traditional approach. However, it is still within the desired Turkish effluent standards (class-B) for irrigation [20].

Table 2

Results with the Optimization Model

Q_i	N_i	$(BOD_5)_i$	T_{avg}
214.80	10000000	340	10.2
Facultative Pond		Maturation Pond	
$(D_T)_f$	$(BWs)_f$	$(D_T)_m$	$(BWs)_m$
40.19	5	20	4
Output			
Facultative Pond			
X	d_f	a	L
76	0.0130	1.16	131.39
W	Ne	Q_e	$(BOD_5)_e$
43.80	22923	184.30	54
Area	V_{conc}		
5754.7691	1045.52		
Maturation Pond			
X	d_f	a	L
34	0.02912879	1.28	84.16
W	Ne	Q_e	$(BOD_5)_e$
43.80	190	164.76	14
Area	V_{conc}		
3685.99	626.63		
Total	Area	D_T	V_{conc}
	9440.76	60.19	1672.16

3.3 Sensitivity analysis

The objective function is proved to be nonlinear since the connection between the variables is not proportional. It can also be observed that the model is sensitive to changes in variables since the longer the retention time in FP, the higher the volume of concrete. Therefore, the model is consistent with reality since if the variables are increased, the area required to contain the wastewater increases, followed by the increase in the required volume of concrete.

To validate the sensitivity, a variation of plus and minus 10 percent in the primary parameters was investigated [23]. A tornado graph was constructed to see which parameters were the most vulnerable to change. Fig. 4 illustrates the sensitivity analysis that was performed on the volume of concrete. As can be observed, the largest bar represents the inflow (Qi) of WSPs. This characteristic is most susceptible to change; the greater the volume, the more the land area needed and the greater the volume of the concrete. The D_T determines the pond system's size following in significance. The least effecting parameter is the number of BWs for the volume of concrete.

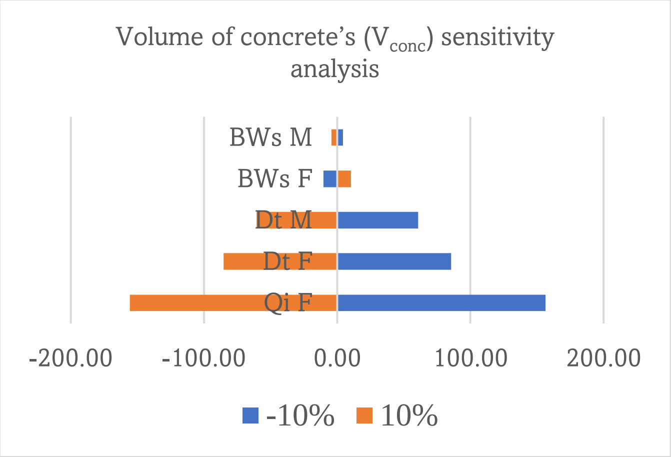


Fig. 4. Volume of concrete's (V_{conc}) sensitivity analysis

Table 3

Percentage reduction in Area, D_T, and V_{conc}

Area	D_T Total	V_{conc}
11.55	13.79	7.19
	Facultative Pond	
19.32	19.32	13.15
	Maturation Pond	
-4.13	0.00	-4.81

Fig. 5 demonstrates that the T_{avg} is the most sensitive parameter influencing the removal of fecal coliforms, i.e., the lower the T_{avg}, the lower the removal of fecal coliforms. The second sensitive parameter is D_T; the shorter the D_T, the lower the removal. The number of fecal coliforms has a negative impact, i.e., the higher the concentration, the lower their removal. Lastly, the inflow (Qi) has no impact on the removal of fecal coliforms.

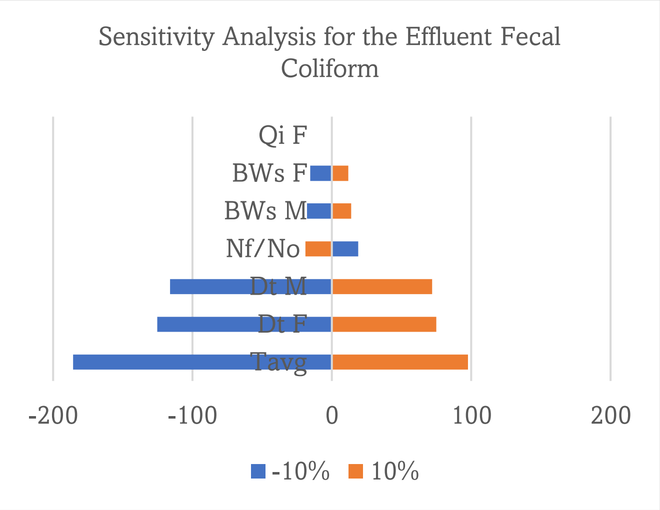


Fig. 5. Sensitivity analysis for Ne (effluent fecal coliforms)

Regarding the biochemical oxygen demand in the effluent of the pond system, Fig. 6 demonstrates that the T_{avg} is the most sensitive parameter, followed by the influent BOD. This indicates that the concentration of effluent organic matter increases with decreasing temperature. The organic matter removal efficiency has upper and lower temperature limitations of 37 and 4 degrees, i.e., beyond this range, the oxygen-producing activity of algae diminish substantially [24]. The D_T takes the following order of importance: the bar shows that the more the organic matter concentration, the shorter the D_T. As can be seen, the sensitivity analysis reacts appropriately to variations in the concept-specific factors. Lastly, as in the case of the fecal coliforms, the inflow (Qi) has no impact on the removal of fecal coliforms.

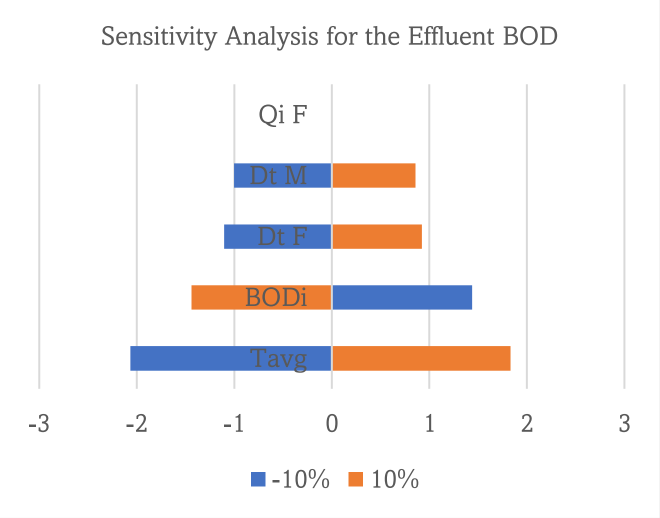


Fig. 6. Sensitivity analysis for the BODe (effluent BOD)

4. Conclusion

According to the aim of this research, using the nonlinear programming, mathematical model yielded superior results: reduced V_{conc}, D_T, and area. It is crucial to note that the current mathematical model may be used to varied design circumstances, i.e., to any location. However, certain modifications must be made, including inflow, temperature, evaporation, BWs, and the materials to be used, among others. In the mathematical model, the effluent BOD_5 from MP was increased from 12 mg/l to 14 mg/l but it is still within the desired limits. However, looking at the Ne, the value reduced from 200 to 190. In both cases, it meets the desired effluent standards.

Nonlinear programming is advised as an alternative supplementary method for optimizing the design of stabilization ponds in developing nations. It achieves significant savings and complies with all of the desired design and effluent standards. It is wise to do further analyses, including an anaerobic pond, and then compare the findings to determine the least area and concrete volume for the implementation of the project.

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