

Numerical study on a heat transfer model in the solid-carbon/liquid-copper systemAttrra Ali ^{*}, Khurram Iqbal*Department of Mathematics, College of Computer Science and Information Systems, Institute of Business Management, Karachi, Pakistan*^{*} Corresponding author: Attrra Ali, Email: attra.ali@iobm.edu.pk

Received: 20 January 2021, Accepted: 13 September 2021, Published: 01 July 2022

KEY WORDS

Thermal Conductivity
Heat Flux
Composites
High-Performance Devices
Pattern
Fourier Law

A B S T R A C T

The thermal conductivity of solid carbon and liquid copper-silicon by using Fourier law, can be utilized as a viable technique to grow superior warm administration materials. The investigation reveals that the thermal conductivity of carbon/copper (C/Cu) composites changes because of the silicon carbide (SiC) relocation from the matrix to the interface. C/Cu composites were used to predict the value of heat flux at different thicknesses of SiC materials. Several studies on heat transfer coefficients in the literature have been reported. However, most of them are experimental reports and only a little fundamental theoretical work have been reported. The possible reasons for this gap might arise from the difficulties of studying the high-temperature and vacuum conditions. A model for the numerical simulation of heat transfer in the solid-carbon/liquid copper-silicon system is presented.

1. Introduction

Effective thermal management is a basic and essential factor considered for designing high efficiency intensive microelectronic chips that empower a wide range devices such as radar, satellites, batteries and other electronics [1]. Rapid technological advancements in recent years have resulted in development of high power devices with reduced dimensional size [2]. Additionally, many electronic devices generate relatively more heat energy when they are in active use compared to their standby condition. Thus, the conventional positive air-cooling technique has become relatively inefficient, and demand for designing novel thermal management techniques is on rise to improve the life and performance of these electronic devices [2, 3]. Moreover, in electronic devices, the heat is generally transferred by conduction mode of heat transfer. High thermal conductivity, low

coefficient of thermal (CTE), and decent machinability are the key parameters to be considered for designing an efficient heat sink [2, 4-6]. Thus, materials offering these features are of great significance for designing efficient thermal management devices for a range of applications [4-8].

Copper (Cu), best known for its excellent conductivity, is a considerably low cost material with good machinability [9]. For that reason, it is believed to be an ideal material for designing the thermal management devices. However, its relatively lower mechanical strength hinders its wider acceptability. Therefore, researchers have been trying to design composites materials, i.e. a group of different materials bonded together by either physical or chemical means, to develop high strength and efficient thermal management devices [10-12]. For instance, various thermal conductive fillers, such as Aluminium (Al),

silver (Ag), etc. has been used as fillers in the polymer matrix to fabricate different composite materials for enhanced thermal management [13].

Moreover, it has been reported that composites materials containing Cu tend to offer relatively better thermal management performance. Furthermore, researchers have been particularly more interested and focused on copper/silicon carbides (Cu/SiC) and copper/diamond composites owing to their high strength and reasonable conductivity [14-15]. Nevertheless, poor machinability of Cu/diamond and relatively low thermal conductivity of Cu/SiC limit their acceptability on mega scale applications in the field thermal management. Therefore, material scientists are pressed on designing and developing new materials that offer both high conductivity, better machinability, superior mechanical characteristics and longer durability [2, 3, 16-18].

Carbonaceous materials are gaining high attention these days owing to their small diameter, high surface area, high mechanical strength, decent electrical conductivity, thermal stability and light weight [19-21]. They have been widely used in composites materials to gain desired dimensional stability and firm thermal properties. Thus, designing composite material that combines carbon and Cu characteristics can potentially be an ideal material with enhanced conductivity and better machinability [22-23].

Herein, numerical study on a heat transfer model in the solid-carbon/liquid-copper system is presented. These types of composite materials are usually fabricated by using liquid melt infiltration (LMI) technique [14]. Factors affecting the thermal management performance of the fabricated composite materials were evaluated through mathematical modelling. Fourier law was used to analyse the heat transfer through the fabricated composite materials.

2. Mathematical Modelling

The measurement of thermal movement at a favourable rate condition can be made to generate heat transfer. Fourier law expresses that the time pace of thermal move through a material is relative to the negative slope in the temperature and the zone, at right points to that inclination, through which the heat streams. Heat opposition is proportional to heat conductance. Similarly, as an electrical opposition is related to the conduction of power, a heat obstruction may be related to the conduction of heat [24, 25]. Heat conduction can be expressed by Eq. 1.

$$\text{Heat flow} = \frac{\text{Thermal potential difference}}{\text{Thermal Resistance}} \quad (1)$$

Where, the thermal potential difference expressed as temperature change (ΔT), i.e, the difference among in and out temperature of the system, where thermal resistance is heating characteristic. Thermal resistance can further be calculated by using Eq. 2.

$$\text{Thermal resistance} = \frac{L}{kA} \quad (2)$$

Where, L , k and A are thicknesses, thermal conductivities and area of plates, respectively.

Fourier Law state that ‘The time rate of heat transfer through a material is proportional to the negative gradient in the temperature and to the area’.

The Fourier law can be mathematically represented by Eq. 3.

$$q = -k\Delta T \quad (3)$$

Where, ΔT , k and q are temperature gradient material conductivity and density of heat flux, respectively.

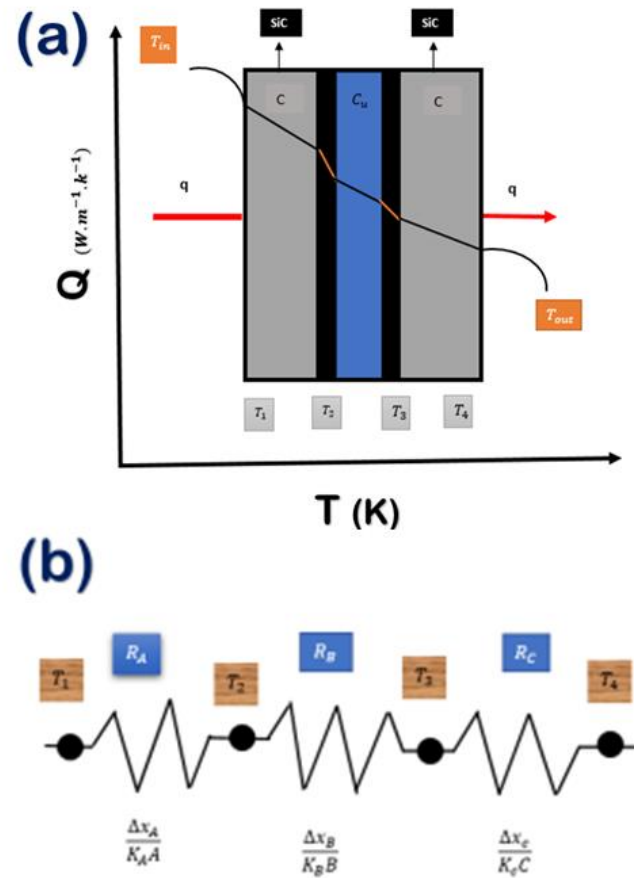


Fig. 1. (a) Schematic diagrams of heat transfer in the solid-carbon/ liquid copper-silicon systems and (b) thermal resistance of all three materials

Fig. 1(a) explains the overview of the calculation in which three layers are joined, SiC is used as epoxy to join carbon fibre with copper composite. Fig. 1(b) shows the thermal resistance of the material during heat

transfer. In this research work, we focused to use Fourier law to calculate heat transfer in solid carbon/liquid-copper system, in this manner, consider a system in which heat passes through three plates, having solid carbon and liquid copper. Small quantity of silicon alloy (SiC) is used to join carbon and copper. For this purpose, initially expanded Fourier law into three basic parts is defined as Eqs. 4-6.

$$q_{cu} = -k_A A \frac{T_2 - T_1}{\Delta x_A} \quad (4)$$

$$q_C = -k_B A \frac{T_3 - T_2}{\Delta x_B} \quad (5)$$

$$q_{SiC} = -k_C A \frac{T_4 - T_3}{\Delta x_C} \quad (6)$$

Eq. 4 represent heat flux passes through copper material, the difference of temperature representing the thermal potential of the material and Δx_A show the thickness of the material. Eq. 5 representing the heat flux that passes through carbon fibre, the thermal potential of carbon fibre is the change in temperature and Δx_B is the thickness of the carbon fibre. Eq. 7 expresses heat flux quantities for silicon carbide with Δx_C thickness. Thus, adding all three equations. Eqs. 4-6 get a combined equation as Eq. 7. Whereas, the Eq. 8 is the final equation for calculating heat flux for overall system.

$$q = \frac{T_2 - T_1 + T_3 - T_2 + T_4 - T_3}{-\frac{\Delta x_A}{k_{AA}} - \frac{\Delta x_B}{k_{BA}} - \frac{\Delta x_C}{k_{CA}}} \quad (7)$$

$$q = \frac{T_1 - T_4}{\frac{\Delta x_A}{k_{AA}} + \frac{\Delta x_B}{k_{BA}} + \frac{\Delta x_C}{k_{CA}}} \quad (8)$$

3. Results and discussion

In this research work, the numerical study of heat transfer was calculated by using Fourier law. There are several materials that have been discovered to get high thermal conductivity because as time passed copper material is no longer durable and having low mechanical strength. Our first approach was to select the combination of that material which fulfil both require high thermal conductivity and high mechanical strength.

Now the value for heat flux in equation (8) which represents the heat flux of the material, these values are helpful to generate a table for individual values of heat flux for thickness of copper and silicon carbide composite. Now our focused is to identify the best values for heat flux and at which consideration of thickness is matter.

In Table 1, Area of solid carbon, liquid copper and silicon carbide was fixed which is one meter square. And heat conductivity of copper was 401 watt per meter-kelvin, heat conductivity of carbon is 1000 watt per meter-kelvin and heat conductivity of silicon is 950 watt

per meter-kelvin. Fixed thickness of copper composite to 5 microns, fixed thickness of carbon is 10 micron, whereas the thickness of silicon carbide thickness was varied from 0.25 micron to 1 micron. Later, the heat flux at thermal difference from 10 to 40°C [26]. Thickness of copper composite will identify the best result for heat flux, which means started from 5 μm thickness of copper, gradually decrease the thickness and at each micron thickness all four values of silicon carbide would use to identify heat flux greatest value.

Table 1

Changes of heat flux at different thicknesses of Silicon Carbide Δx_c with fix thickness of solid carbon ($\Delta x_a = 10(\mu\text{m})$) and liquid copper ($\Delta x_b = 5(\mu\text{m})$)

	ΔT (K)				
	10	20	30	40	
Δx_c	0.25	439.909	879.818	1319.73	1759.64
(μm)	0.5	434.874	869.749	1304.62	1739.5
	0.75	429.954	859.908	1289.89	1719.82
	1	425.144	850.287	1275.43	1700.57

Table 2 defines the numerical values for heat flux. Area of solid carbon, liquid copper and silicon carbide is fixed which is one meter square. And heat conductivity of copper is 401 watt per meter-kelvin, heat conductivity of carbon is 1000 watt per meter-kelvin and heat conductivity of silicon is 950 watt per meter-kelvin. fixed thickness of copper composite to 4 microns, fixed thickness of carbon is 10 microns [26]. It has been noticed that thickness of silicon carbide as increased which result to decreases the heat flux, that means increases in temperature would increase heat flux, but the thickness of the silicon carbide opposed this relation.

Table 2

Changes of heat flux at different thicknesses of Silicon Carbide Δx_c with fix thickness of solid carbon ($\Delta x_a = 10(\mu\text{m})$) and liquid copper ($\Delta x_b = 4(\mu\text{m})$)

	ΔT (K)				
	10	20	30	40	
Δx_c	0.25	494.115	988.229	1482.34	1976.46
(μm)	0.5	481.59	963.181	1444.77	1926.36
	0.75	487.772	975.544	1463.32	1951.09
	1	475.563	951.127	1426.69	1902.25

Table 3 is the mathematical iteration for heat flux by using Eq. 8. Table 3 defines the numerical values for heat flux. Area of solid carbon, liquid copper and silicon carbide is fixed which is one meter square. And heat conductivity of copper is 401 watt per meter-kelvin, heat conductivity of carbon is 1000 watt per meter-kelvin and

heat conductivity of silicon is 950 watt per meter-kelvin. fixed thickness of copper composite to 3 microns, fixed thickness of carbon is 10 microns [26]. In table 3 our consideration is the same as used in previous table.

Table 3

Changes of heat flux at different thicknesses of Silicon Carbide Δx_c with fix thickness of solid carbon ($\Delta x_a = 10(\mu m)$) and liquid copper ($\Delta x_b = 3(\mu m)$)

		ΔT (K)			
		10	20	30	40
Δx_c (μm)	0.25	563.556	1127.11	1690.67	2254.23
	0.5	547.322	1094.64	1641.97	2189.29
	0.75	555.32	1110.64	1665.96	2221.28
	1	539.551	1079.1	1618.65	2158.2

Table 4 is the last iteration for thickness value of the copper composite, i.e., 2 microns. Table 4 defines the numerical values for heat flux. Area of solid carbon, liquid copper and silicon carbide is fixed which is one meter square [26]. And heat conductivity of copper is 401 watt per meter-kelvin, heat conductivity of carbon is 1000 watt per meter-kelvin and heat conductivity of silicon is 950 watt per meter-kelvin. Tables 1, 2, 3 and 4 provides the different values of heat flux to optimize the thickness of copper composite with respect to change in thickness of silicon carbide.

Table 4

Changes of heat flux at different thicknesses of Silicon Carbide Δx_c with fix thickness of solid carbon ($\Delta x_a = 10(\mu m)$) and liquid copper ($\Delta x_b = 2(\mu m)$)

		ΔT (K)			
		10	20	30	40
Δx_c (μm)	0.25	655.708	1311.42	1967.12	2622.83
	0.5	633.834	1267.67	1901.5	2535.34
	0.75	633.834	1267.67	1901.5	2535.34
	1	623.435	1246.87	1870.31	2493.74

Fig. 2 represents the graphical interpretation of heat flux, graph is plotted between heat flux and temperature, heat flux is drawn vertically and temperature is drawn horizontally, as this was the combine result of all three materials in which copper and carbon thickness is fixed and the thickness of silicon carbide varied. Results clearly show that with increments in temperature, thermal conductivity also increased.

In Fig. 3, the straight line represents the relationship between temperature and heat flux which is showing that the heat flux is increasing as the thickness of copper composite decreased. Blue line is the value of heat flux

at 0.25 micron thickness of silicon carbide, where thickness of carbon fixed at 10 micron and thickness of copper is fixed at 4 microns. The comparison between Fig. 3 and Fig. 4 is indicating the heat flux value increased.

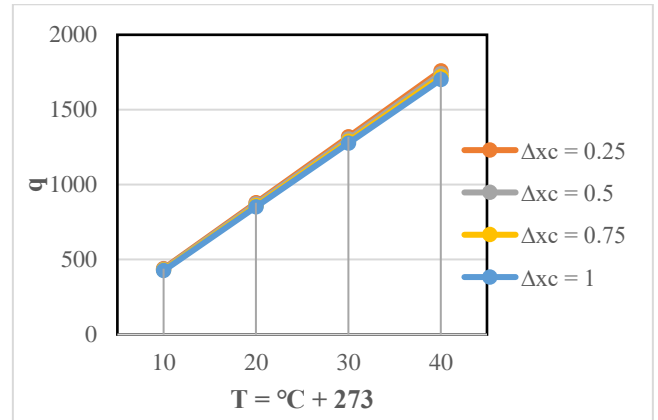


Fig. 2. Graphical presentation between heat flux and temperature at $\Delta x_a = 5 \mu m$, thickness of copper composite.

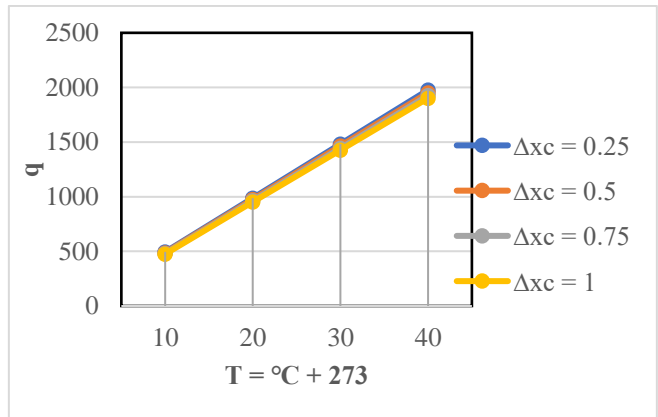


Fig. 3. Graphical presentation between heat flux and temperature at $\Delta x_a = 4 \mu m$, thickness of copper composite.

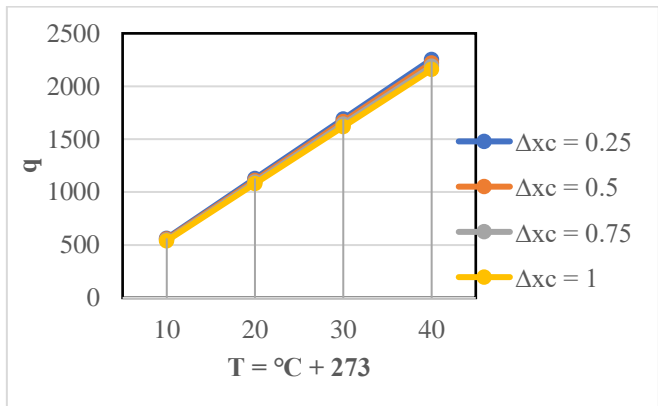


Fig. 4 Graphical presentation between heat flux and temperature at $\Delta x_a = 3 \mu m$, thickness of copper composite.

Fig. 4 is the result of Table 3, in this figure it has been noticed that the value of the heat flux increased with time. Moreover, when compared the result of Fig. 5 with previous Fig. 4, it is clearly observed that the heat flux

in increasing as decreased the thickness of copper composite.

In Fig. 5, temperature and heat flux showing stream line though the two-dimensional axis, it is noticeable that heat flux is giving a maximum value at temperature difference 40°C, with increasing the thickness of silicon carbide it has been noticeable that heat flux decreased. Figs. 2, 3, 4 and 5 are respectively the graphical representation of heat flux q and thermal difference ΔT .

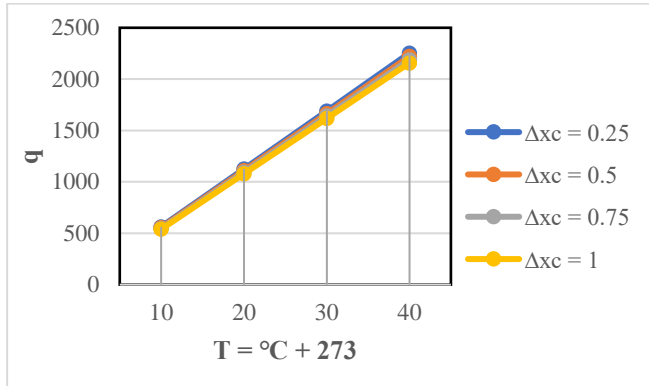


Fig. 5 Graphical presentation between heat flux and temperature at $\Delta x_c = 2 \mu m$, thickness of copper composite.

4. Conclusion

In this research work the effective numerical study about thermal conductivity of solid carbon/liquid copper was carried out using silicon carbide as thermal interface between carbon and copper. The objective of this research was to use the positive results from the previously reported works and apply them to a different material system to develop a composite system with superior thermal properties. It is observed that increase in thickness of silicon carbide was the reason to increase the thermal conductivity of the system, whereas the increment in the carbon content decreased the thermal conductivity. Thus, it was concluded that the critical factor for enhanced thermal conductivity was the thickness of the material.

5. References

- [1] Q. Li, C. Yang, S. Santhanagopalan, K. Smith, J. Lamb, L. A. Steele, and L. Torres-Castro, "Numerical investigation of thermal runaway mitigation through a passive thermal management system", *Journal of Power Sources*, vol. 429, pp. 80-88, 2019.
- [2] Y. H. Kim, D. K. Lee, H. G. Cha, C. W. Kim, Y. C. Kang, and Y. S. Kang, "Preparation and characterization of the antibacterial Cu nanoparticle formed on the surface of SiO₂ nanoparticles", *The Journal of Physical Chemistry B*, vol. 110, pp. 24923-24928, 2006.
- [3] L. M. Davis and D. W. Thompson, "Novel and facile approach to the fabrication of metal-patterned dielectric substrates", *Chemistry of Materials*, vol. 19, pp. 2299-2303, 2007.
- [4] C. Zweben, "Advances in composite materials for thermal management in electronic packaging", *JOM*, vol. 50, pp. 47-51, 1998.
- [5] P. K. Schelling, L. Shi, and K. E. Goodson, "Managing heat for electronics", *Materials Today*, vol. 8, pp. 30-35, 2005.
- [6] W. Buchgraber, G. Korb, T. Schubert, and B. Kempf, "Carbon fibre reinforced copper matrix composites: Production routes and functional properties", *Microstructural Investigation and Analysis*, vol. 4, pp. 150-155, 2000.
- [7] S. Berber, Y.-K. Kwon, and D. Tománek, "Unusually High Thermal Conductivity of Carbon Nanotubes", *Physical Review Letters*, vol. 84, pp. 4613-4616, 2000.
- [8] R. C. Chu, R. E. Simons, M. J. Ellsworth, R. R. Schmidt, and V. Cozzolino, "Review of cooling technologies for computer products", *IEEE Transactions on Device and Materials Reliability*, vol. 4, pp. 568-585, 2004.
- [9] A. Rape, "Copper matrix composites fabricated via field assisted sintering technology for thermal management applications", Ph.D. Thesis, The Pennsylvania State University, 2015.
- [10] Y. Sun, L. He, C. Zhang, Q. Meng, B. Liu, K. Gao, M. Wen, and W. Zheng, "Enhanced tensile strength and thermal conductivity in copper diamond composites with B 4 C coating", *Scientific Reports*, vol. 7, pp. 1-10, 2017.
- [11] T. Schubert, W. Zieliński, A. Michalski, T. Weißgärber, and B. Kieback, "Interfacial characterization of Cu/diamond composites prepared by powder metallurgy for heat sink applications", *Scripta Materialia*, vol. 58, pp. 263-266, 2008.
- [12] D. Zhu, W. Yu, H. Du, L. Chen, Y. Li, and H. Xie, "Thermal conductivity of composite materials containing copper nanowires", *Journal of nanomaterials*, vol. 2016, pp. 1-6, 2016.
- [13] L.-C. Jia, Y.-F. Jin, J.-W. Ren, L.-H. Zhao, D.-X. Yan, and Z.-M. Li, "Highly thermally conductive liquid metal-based composites with superior thermostability for thermal management", *Journal of Materials Chemistry C*, vol. 9, pp. 2904-2911, 2021.

- [14] K. Iqbal, J. J. Sha, Z.-K. Lei, A. Maqsood, and M. Mujahid, "Numerical studies of infiltration dynamics of liquid-copper and silicon/solid-carbon system", *JOM*, vol. 66, pp. 953-959, 2014.
- [15] K. Iqbal, "Numerical studies of infiltration dynamics of liquid-copper and titanium/solid-carbon system", *Processing and Properties of Advanced Ceramics and Composites VII: Ceramic Transactions, Volume 252*, pp. 157-167, 2015.
- [16] K. Woo, D. Kim, J. S. Kim, S. Lim, and J. Moon, "Ink-Jet printing of Cu-Ag-based highly conductive tracks on a transparent substrate", *Langmuir*, vol. 25, pp. 429-433, 2009.
- [17] N. Balachander, I. Seshadri, R. J. Mehta, L. S. Schadler, T. Borca-Tasciuc, P. Keblinski, and G. Ramanath, "Nanowire-filled polymer composites with ultrahigh thermal conductivity", *Applied Physics Letters*, vol. 102, p. 093117, 2013.
- [18] S. Kumar, A. Kumar, R. Devi, A. Shukla, and A. Gupta, "Capillary infiltration studies of liquids into 3D-stitched C-C preforms: Part B: Kinetics of silicon infiltration", *Journal of the European Ceramic Society*, vol. 29, pp. 2651-2657, 2009.
- [19] S. Wu, T. Yan, Z. Kuai, and W. Pan, "Thermal conductivity enhancement on phase change materials for thermal energy storage: A review", *Energy Storage Materials*, vol. 25, pp. 251-295, 2020.
- [20] T. Schubert, A. Brendel, K. Schmid, T. Koeck, W. Zielinski, T. Weißgärber, and B. Kieback, "Interfacial design of Cu/SiC composites prepared by powder metallurgy for heat sink applications", *Composites part A: Applied science and Manufacturing*, vol. 38, pp. 2398-2403, 2007.
- [21] K. Chu, Z. Liu, C. Jia, H. Chen, X. Liang, W. Gao, W. Tian, and H. Guo, "Thermal conductivity of SPS consolidated Cu/diamond composites with Cr-coated diamond particles", *Journal of Alloys and Compounds*, vol. 490, pp. 453-458, 2010.
- [22] R. Asthana, "Dynamic Wetting Effects During Infiltration of Metals", *Scripta Materialia*, vol. 38, pp. 1203-1210, 1998.
- [23] Y. Liu, C. Zhang, S. Qiao, and Z. Yang, "Fabrication and microstructure of C/Cu composites", *Advanced Engineering Materials*, vol. 12, pp. 493-496, 2010.
- [24] D. Kumlutaş, I. H. Tavman, and M. T. Çoban, "Thermal conductivity of particle filled polyethylene composite materials", *Composites Science and Technology*, vol. 63, pp. 113-117, 2003.
- [25] I.-S. Liu, "On Fourier's law of heat conduction", *Continuum Mechanics and Thermodynamics*, vol. 2, pp. 301-305, 1990.
- [26] Y. Agari and T. Uno, "Estimation on thermal conductivities of filled polymers", *Journal of Applied Polymer Science*, vol. 32, pp. 5705-5712, 1986.