Mehran University Research Journal of Engineering and Technology

https://doi.org/10.22581/muet1982.2203.09

2022, 41(3) 87-96

Guaranteed consensus performance in second-order networked multi-agent systems with induced delay

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Received: 22 July 2020, Accepted: 30 November 2021, Published: 01 July 2022

KEYWORDS

Multi-agent Systems

Second-order System

Complementary Sensitivity Function

Controller Parameterization Induced Delay

Frequency Domain

ABSTRACT

This work described the performance of consensus control of the second-order networked multi-agent system with CAN bus-induced delay. First, the characteristics of controller area networks (*CAN*) buses and the main causes of CAN bus induce a delay in distributed networked control systems (*DNCS*) were analysed. Secondly, *DNCS* is converted into a model of networked multi-agent systems in the frequency domain. Then based on the designed model of networked multi-agent systems, an analytical H_2 controller is proposed for performance tracking. Moreover, a complementary sensitivity function is used to assess the robustness of the proposed controller. In the end, delay margin criteria are derived for the consensus of a networked multi-agent system in the presence of CAN bus-induced delay. The results obtained by simulating the second-order networked multi-agent system verified the usefulness of the proposed control protocol.

1. Introduction

A networked multi-agent system (NMAS) contains agents which coordinate with each other through a shared communication medium to perform tasks. From last decade, it has been observed that the NMASs have gained a tremendous attraction because of their applications in the areas of network control systems [1], DC microgrids [2,3],multi-robots [4,5], formation control of unmanned Arial vehicles [6], mobile robots [7] and distributed sensors networks[8]. A common objective of all these works is to propose a control scheme that makes agents reach a single agreement point (called consensus) in the existence of uncertainties, disturbances, and time-varying delays. Many factors like the attack on the system and time delays can disturb the performance of NMAS. Among these, time delays affect the system badly which may result in loss of consensus.

A lot of work has been presented to address the problems of time delays which often occur in real applications and influence their performance. There can be two types of delays in NMASs, input time delay (T_i) and communication time delay (T_c). T_i is because of the internal processing of an agent whereas T_c occurs in the communication of two agents. In some real applications, the input time delay may be called computational delay or CAN bus induced delay [9,10]. Further, there is a congested communication medium and limited bandwidth for the information flow which can cause CAN bus induce a delay between agents as well [11]. This CAN bus induce delay is not favourable because it destroys the system’s consensus, stability, and performance.

Many techniques have been proposed to minimize the CAN bus induce delay effects. For example, an analytical controller is designed in the frequency domain for the consensus control of NMAS in the presence of CAN bus induce delay [12]. After that, a linear model based on internal stability analysis is proposed to solve the issue of CAN bus induce delay and communication delay in NMASs [13]. Later, an estimation control technique for non-linear NMAS is proposed to achieve consensus in the existence of CAN bus induce delay [14]. After that, a tracking control scheme is designed for the CAN bus induce delay issue of nonholonomic agents [15]. Moreover, a linear dynamic model based on graph theory is proposed for the communication delay problem of second-order NMASs [16]. Later, a PID control scheme is proposed to achieve the synchronous consensus of MAS in the frequency domain [17]. After that, a linear framework for a multi-agent system is designed to improve the H_2 and H_∞ performance in the presence of delays [18-20]. Later, consensus conditions for performance of multi-agent systems with changeable convergence speed and CAN bus induce delay discussed by [21-25].

There has been a lot of research done were delays in multi-agent systems considered and analysed [12-14]. These works have discussed both T_i and T_c either individually or combined. In some literature, T_i has been studied as time-varying or multiple input time delays. But the effects of CAN bus induced delay are not considered in most of the works [11]. In NMAS, controllers, sensors, and actuators of agents are connected through the CAN bus which may cause a delay in information exchange [26]. This CAN bus induce delay can make the system unstable. Thus, there is a need for a consensus control technique for the robust performance of NMAS.

From the literature, the frequency domain has become a famous tool for designing controllers because the computational cost of the frequency domain is lower. Similarly, any controller can be easily analysed because frequency domain tools are easily available, and designing is less complex. Therefore, the frequency domain tool can be used effectively to design the consensus controller of NMAS [26].

Our focus is to propose a consensus control protocol to guarantee the performance criteria under the influence of networked CAN bus induced delay. We are applying consensus conditions for the networked multi-agent system with a dynamical second-order model for the agents with CAN bus induced delay. In the start, we obtain consensus conditions for delay-free (where no delay is introduced in the system) case. After obtaining the consensus condition for a delay-free case, we formulated a formula to achieve consensus for time delay by analysing the connection between the roots of characteristic equations and time delay frameworks. From this, it is devised that there exists a balance between robustness and consensus performance. The contribution of this paper is summaries as below.

1. First of all, we analysed the characteristics of CAN bus and traced the main causes of induced delay in CAN buses of distributed networked control systems.
2. A model of networked multi-agent systems is designed in the frequency domain by converting Distributed networked control systems.
3. Then based on this designed model, an analytical H_2 controller is proposed for the performance index using the controller parameterization technique.
4. Complementary sensitivity function is used for performance tracking and robustness of the controller.
5. Delay margin criteria is derived between the coordinating agents using stability function.

Remaining article is organized as, section 2 presents the basic concepts of mathematics and problem formulation, section 3 proposes design of robust control by using internal stability analysis, section 4 describes the simulation results of test case, and section 5 finally concludes the research work.

1.1 Definition 1

Cyber-physical systems are also called intelligent systems in which computer algorithms are used to control the system mechanisms.

1.2 Definition 2

Controller Area Network (CAN) is a protocol that agents use to coordinate with each other without central a point. All agents can send or receive information sequentially, but priority is assigned to every agent. Due to the priority, any agent may have to wait for a short period causing the CAN bus to induce delay.

1.3 Definition 3

Multi-agent systems connected through a network are called distributed network control systems (DNCS).

2. Mathematical Preliminaries and Problem Formulations

2.1 Assumption: Directed and Fixed Communication Topology of NMAS

Let us consider a homogenous networked multi-agent system (MAS) [12] which consists of *n* agents. Each agent of MASs have the same transfer function i.e. $F_1(s) = F_2(s) = F_3(s) = \cdots F_n(s)$ and they have identical controllers i.e. $Co_1(s) = Co_2(s) = \cdots Co_n(s)$. The communication topology of agents is fixed and directed. Moreover, all agents are classified into two sets. Some agents (m) are connected directly with equal external reference i.e $E_{ri1}(s) = E_{ri2}(s) = E_{ri3}(s), \ldots = E_{rim}(s) = E_r(s)$ while rest of the agents (*n-m*) don't have direct access to external reference i.e. $E_{ri1}(s) = E_{ri2}(s) = E_{ri3}(s), \ldots = E_{rim-n}(s) = E_r(s) = 0$. From Fig. 1, output state should be equal to the reference input. Therefore, by using Eq. 1, dynamic of each sub-system of MASs can be defined.

$$X_i(s) = F_i(s)U_i(s) = F_i(s)C_o(s)E_i(s) \quad (1)$$

Where *F*(*s*) is the transfer function of the subsystem, $C_o(s)$ is the controller, Sate of the subsystem is $X_i(s)$, *U(s)* is control input or reference input and E_i(s) is the state error. Communication between the agents take place and through communication channel exchange of information is done. Due to which relative state error between *m* subsystem can be defined as Eq. 2.

$$E_i(s) = E_{ri}(s) + X_i(s) + \sum_{j \in Ni} a_{ij}[X_j(s) - X_i(s)]; j = 1,2,3..m \quad (2)$$

Similarly, *n-m* subsystems which are not directly connected to external reference input i.e. $E_r = 0$, therefore, relative state error can be written as follows.

$$E_i(s) = \sum_{j \in N_i} a_{ij}\left[X_j(s) - X_i(s)\right];\ j = m+1, m+2, m+3\ldots m-n \quad (3)$$

In vector form, it is written as follow.

$$E_i(s) = E_r(s) - I_m^n X_i(s) - LX_i(s) \quad (4)$$

Where L and I are the Laplacian and identity matrices, respectively. Therefore, based on the above analysis, this considered MAS can be converted into a closed-loop MIMO system as shown in Fig. 1. We may calculate the output of the MASs in vector form as $X_i(s) = [X_1(s), X_2(s), \ldots. X_n(s)]^T$. All the controllers are also identical. $Z_{ei}(s) = [Z_{e1}(s), Z_{e2}(s) \ldots. Z_{en}(s)]^T$is the external input to the system and consensus values are $E_r(s) = [E_{r1}(s), E_{r2}(s) \ldots. E_{rn}(s)]^T$. Hence, we may represent the transfer function and controller values of the closed-loop MAS by $F(s) = \oplus \sum_{i=1}^{N} F_i(s)$, $C_o(s) = \oplus \sum_{i=1}^{N} C_{oi}(s)$.

Let $D_{i\text{in}}$ and $D_{i\text{out}}$ be the inputs and outputs disturbance vectors respectively as shown in Fig. 1. A system error between the consensus value $E_r(s)$ and external input $Z_e(s)$ while information sharing among the coordinating agents can be represented as $E_i(s) = [E_1(s), E_2(s) \ldots, E_n(s)]$. Information exchange between the agents can be calculated by $L + I_m^n$. Therefore, we may represent MAS as $L + I_m^n$??

$$X(s) = C_o(s)F(s)[I + (L + I_m^n)C_o(s)F(s)]^{-1}E_r(s).$$

The primary concern that needs to be considered in the design of the control system is that in case of system disturbance and uncertainty, the error should be small as possible between the output and input of the system. The point tracking and disturbance rejection/uncertainty is calculated by the H_2 performance index. Due to the optimal H_2 performance of networked multi-agent systems, we may minimize the output rejection level. H_2 performance index of the system can be written as Eq. 5.

$$min \int_0^\infty s^t(t)s(t)dt = min||s(t)||_2^2 \quad (5)$$

Sensitivity of the system can be expressed by the effective output uncertainty to output state, if we consider that model is exact as Eq. 6.

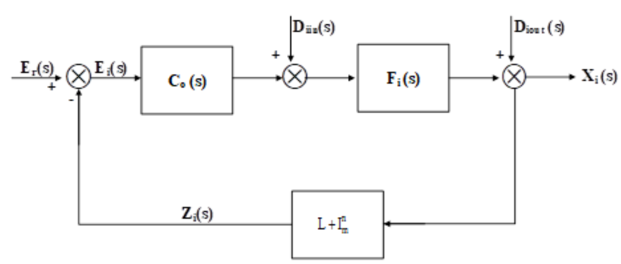

Fig. 1. MIMO framework of MAS

$$S_n(s) = X(s)/D_{iout} = E_r(s)/E_i(s) = 1/([I_m^n + L'F'(S)C_o'(s)]) \quad (6)$$

Where $L' = L_a + I$.

Suppose $F_c(s)$ denotes the complimentary transfer function. Then we write it as Eq. 7.

$$F_c(s) = X(s)/E_r(s) = (F'(s)C_o'(s))/[I_m^n + L'F'(s)C_o'(s)] \quad (7)$$

Because of the variations in the transfer function *X(s)*, the sensitivity of complementary transfer function $F_c(s)$ is computed using Eq. 6.

In the existence of linear time-invariant systems, causal and homogeneous agent systems, we aim to determine the conditions for consensus of second-order MAS's and to determine the delay margin between the interacting agents in the MAS. The next section discusses the proposed design of the controller.

3. Design of Proposed Controller

3.1 Controller design

Let us have a dynamical multi-agent system. The transfer function of a dynamical agent can be written as[12]:

$$F(s) = \left(\frac{GN_{m+}(s)N_{m-}(s)}{D_{m-}(s)D_{m+}(s)}\right)e^{(-ds)} \quad (8)$$

Where G and d are real constant gain and induced delay of the system respectively. The + and – symbols denote the right half plane (RHP) and left half plane (LHP) respectively. $N_{m+}(s)$ and $D_{m+}(s)$ denotes the roots in the RHP. $N_{m-}(s)$ and $D_{m-}(s)$ denotes the roots in the LHP. Suppose that $N_{m+}(0) = N_{m-}(0) = D_{m-}(0) = D_{m+}(0) = 1$ and $deg(N_{m+}(s)) + deg(N_{m-}(s)) \leq deg(D_{m+}(s)) + deg(D_{m-}(s))$. These assumptions suggest that the transfer function of agents is in proper form with a constant value of 1. From the previous discussion, we can dissolve our system into 'm' subsystems to analyse the performance of the system. We may define characteristics equation $G_u(s)$ of unity feedback system as Eq. 9.

$$G_u(s) = C_o(s)[1 + \lambda_i C_o(s)F(s)]^{-1} \quad (9)$$

In a parallel path model, the effect of manipulated variable is subtracted from the output process. If we assume that the model represents the process perfectly, then the feedback signal should be equal to the disturbance impact and it should not be in the effect of manipulated variables as shown in Fig. 2. Therefore, in the case of open loop systems, stability problems due to feedback signal are resolved. The system should be called stable only if all the processes and associated controllers are stable. We can explain it in block matrix as Eq. 10.

$$S_i(s) = \begin{bmatrix} F(s)G_u(s) & F(s)[1 - \lambda_i F(s)G_u(s)] \\ G_u(s) & -\lambda_i F(s)G_u(s) \end{bmatrix} \quad (10)$$

Since, each agent may have unstable poles u_v. Unstable pole v_j (j = 1, 2, 3…u_v) is the multiple of I_j, where j = 1, 2, 3,…n, i.e. $D_{e+}(s) = \prod_{j=1}^{uv}(-v^{-1}s + 1)^I$.

Following are the conditions to stabilize the quasi-system.

1. G_u *(s)* must be stable.
2. When there are unstable poles in the $F(s)$, then $G_u(s)$ and $1 - \lambda_i F(s)G_u(s)$ contain zeros.
3. Pole zero cancellations $C_o(s)$ can be removed from Right Half Plane.

From Fig. 2, $S_n(s) = 1 - \lambda_i F(s)G_u(s)$ can be used to describe the effect of external reference input $E_r(s)$ to the error $E_i(s)$. Since, different types of sinusoidal signals are present in the step signal which shows an increase and decrease in amplitude and frequency.

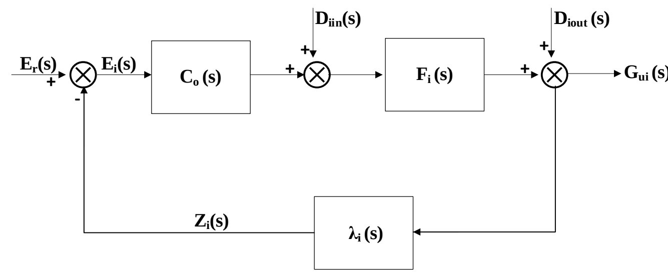

Fig. 2. Block diagram representation of internal model control (IMC) method

That's why we chose step input for sensible analysis. We know that if $t \to \infty$ then in Laplace domain $s \to 0$. Therefore, we may track the step input by using the following Eq. 11.

$$\lim_{s\to 0} S_{ni}(s) = 0 \quad (11)$$

Theorem 1 is presented to analyse the performance of proposed controller.

3.1.1 Theorem 1

Agents are described by the transfer function mentioned in [12]. If they are coordinating with each other through directed or undirected communication topology, then homogeneous MAS achieves consensus subsequently and H_2 performance index achieved can be explained as

$$C_o(s) = \frac{k[D_{m+}(s)D_{m-}(s)]}{GN_{m-}(s)[N_{m+}(-s) - N_{m+}(s)e^{(-ds)}]} \quad (12)$$

Where k is parameter of controller.

3.1.1.1 Proof: Since magnitude of all-pass transfer functions $N_{m+}(s)/\ N_{m+}(-s)$ is 1 and the delay between the agent is $e^{(-ds)}$. Then all negative pass transfer function is not affected by the 2-norm values. Therefore, we may write matrix decomposition as follow.

$$||S_{ni}(s)V(s)||_2^2 = ||1/s[1 - \lambda_i F(s)G_u(s)]||_2^2|| \quad (13)$$

Transfer function of the agents borrowed from T. I. Fossen's work [28] is written as follows.

$$||S_{ni}(s)V(s)||_{2}^{2}=||1/s[1-\frac{(N_{(m+)}(s)N_{(m-)}(s))}{(D_{(m-)}(s))}e^{(-ds)}-\frac{(sN_{(m+)}(s)N_{(m-)}(s))}{(D_{(m-)}(s))}e^{(-ds)}G_b(s)]||_{2}^{2} \quad (14)$$

$$|S_{ni}(s)V(s)||_{2}^{2}=||\frac{(N_{(m+)}(s)e^{(-ds)})}{(N_{(m+)}(-s))}\times[\frac{(N_{(m+)}(-s)e^{ds})}{(sN_{(m+)}(s))}-\frac{(N_{(m+)}(-s)N_{(m-)}(s))}{(sD_{(m-)}(s))}-\frac{(N_{(m+)}(-s)N_{(m-)}(s))}{(D_{(m-)}(s))}G_b(s)]||_{2}^{2}$$

Further simplification gives us the following equation.

$$||S_{ni}(s)V(s)||_{2}^{2}=||\frac{(N_{(m+)}(-s)e^{ds})}{(sN_{(m+)}(s))}-\frac{(N_{(m+)}(-s)N_{(m-)}(s))}{(sD_{(m-)}(s))}-\frac{(N_{(m+)}(-s)N_{(m-)}(s))}{(D_{(m-)}(s))}G_b(s)||_{2}^{2} \quad (15)$$

As we know from previous assumption, i.e. $N_{m+}(0)=N_{m-}(0)=D_{m-}(0)=D_{m+}(0)=1$; we may rewrite the above equation as under.

$$||S_{ni}(s)V(s)||_{2}^{2}=||\frac{(N_{(m+)}(-s)e^{ds}-N_{(m+)}(s))}{(sN_{(m+)}(s))}+\frac{(M_{-}(s)-N_{(m+)}(-s)N_{(m)}(s))}{(sD_{(m-)}(s))}-\frac{(N_{(m+)}(-s)N_{(m-)}(s))}{(D_{(m-)}(s))}G_b(s)||_{2}^{2} \quad \ldots(16)$$

$$||S_{ni}(s)V(s)||_{2}^{2}=||\frac{(N_{(m+)}(-s)e^{ds}-N_{(m+)}(s))}{(sN_{(m+)}(s))}||_{2}^{2}+||\frac{(M_{-}(s)-N_{+}(-s)N_{(s)})}{(sM_{-(s)})}-\frac{(N_{+}(-s)N_{(s)})}{(M_{-(s)})}G_b(s)||_{2}^{2} \quad (17)$$

Simplifying R.H.S, $\frac{D_{m\_}(s)-N_{m+}(-s)N_{m-}(s)}{sD_{m\_}(s)}-\frac{N_{m+}(-s)N_{m-}(s)}{D_{m\_}(s)}G_b(s)=0$.

Optimal value of the transfer function is given as follows.

$$G_{b-op}=\frac{D_{m-}(s)N_{m+}(-s)N_{m-}(s)}{sN_{m+}(s)N_{m-}(s)} \quad (18)$$

$$G_{b-op}=\frac{D_{m+}(s)D_{m-}(s)}{\lambda_i GN_{m+}(-s)N_{m-}(s)} \quad (19)$$

Results of the designed optimal controller for the decomposed system are strictly connected with the values of the Laplacian matrix i.e λ_i. The network structure is under the effect of controller performance and global information λ_i shared between the agents should be known by every agent. For large networks, it is difficult to capture all the information for agents by only one controller. Because of this, we implemented *N* controllers of the same type and structure. And for establishing a standard of configuration and design, one may replace *k* with $1/\lambda_i$.

We may find feasibility as follows.

$$\lim_{s\to 0}[\frac{1}{(F(s)C_o(s))}]=\lim_{s\to 0}[\frac{(N_{(m+)}(-s)e^{ds}-N_{(m+)}(s))}{(kN_{(m+)}(s))}]=0 \quad \ldots(20)$$

The stable response of the system is not under the effect of constant *k* but it helps in tuning the H_2 performance index of the single agent. We may write the transfer function of agents as follows.

$$||S_{ni}(s)V(s)||_{2}^{2}=||\frac{(N_{m+}(-s)e^{ds}-N_{m+}(s))}{(sN_{(m+)}(s))}||_{2}^{2} \quad (21)$$

Put the value of *k* in the above equation, we would have Eq. 22.

$$||S_{ni}(s)V(s)||_{2}^{2}=||\frac{(N_{(m+)}(-s)e^{ds}-N_{(m+)}(s))}{(sN_{(m+)}(-s)e^{ds}+(\lambda_i k-1)sN_{(m+)}(s))}||_{2}^{2} \quad \ldots(22)$$

If the agent is the minimum phase system of a dynamic system, then $G_{b-op}(s)$ can be minimized to $\frac{1}{\lambda_i G_{b-op}(s)}$.

3.1.2 Lemma 1 [12]

The system is stable and gives robust performance for any frequency if and only if $||\Delta_m(s)F_c(s)||_\infty<1$ and $|||V(s)S_n(s)|+|\Delta_m F_c(s)|||_\infty<1$.

The $G_{b-op}(s)$ proposed optimized controller is made proper by using a type 1 filter, as $G_{b-op}(s)$ is normally improper. Type 1 filter can be written as

$$j(s)=\frac{(\partial_n s^n+\ldots+\partial_1 s^1+\partial_0)}{(\gamma s+1)^m} \quad (23)$$

Where γ a positive constant is called the performance degree of the filter. The parameter m_i is chosen to make $G_{i-op}j(s)$ semi-proper. A controller becomes more optimized by the use of filer. A final redesigned controller can be written as Eq. 24.

$$C(s)=\frac{k[D_{(m+)}(s)D_{(m-)}(s)]}{GN_{(m-)}(s)[N_{(m+)}(-s)-N_{(m+)}(s)e^{(-ds)}j(s)]} \quad (24)$$

The above equation shows the balance between robustness and performance. Filter parameter γ controls the amplitude of the control variables. It also provides a balance between robustness and minimal performance. We have minimal performance at $\gamma=0$ but performance increases with the increase in γ value. In the next section, the delay margin is calculated using theorem 2.

3.2 Delay margin criteria

3.2.1 Theorem 2

Assume we have a control protocol that helps MAS achieving consensus for delay free situations. Let $w_i>0$

defines the root of the equation given as $(w^2{}_i+a)^2+b^2w^2{}_i$– $(k^2{}_1+k^2{}_2w^2{}_i)|\lambda_i|^2=0$, for $i=\ 2,3,..n$.

Let $d_i=\{k\pi+tan^{-1}(\Psi_i)\}w^{-1}{}_i$, where d_i is the delay value and $\Psi_r=\frac{\phi_i(w_i+a)+\varphi_i bw_i}{\phi_i bw_i+\varphi_i(w_i+a)}$ with $\phi_i=k_2w_i\,Re(\lambda_i)+k_1\,Im(\lambda_i)$ and $\varphi_i=k_2w_i\,Im(\lambda_i)+k_1\,Re(\lambda_i)$. Also $d_i>0$ for the smallest value of k. We selected the minimum value of delay from all delay values, i.e. $d_i{}'=min$ (all the delay values overall root values). So that MAS could achieve consensus when $d_i\in[0,d_i{}')$ [12].

3.2.1.1 Proof: In the light of the characteristics defined in Eq. 18, assumed that the quasi-polynomial of Eq. 9 is $S(s,e^{-ds})=S_0(s)+S_1(s)\,e^{-ds}$, where $S_0(s)=f^n+a_1f^{n-1}+\cdots.a_n$ and $S_1(s)=\ b_1f^{n-1}+b_2f^{n-2}\ ...b_n$.

By Routh Hurwitz stability criteria, if $S(s,e^{-ds})$ show stability for d = 0 and instability for d > 0 then $S(s,e^{-ds})$ contain poles on an imaginary axis such that $0<d'<$ d and $S(s,e^{-d_0s})$ is stable for all $d^0<d'$. And, multi-agent system achieves consensus for d = 0 then the roots of the equation will be in the LHP for all $d_i\in[0,d_i{}')$ if and only if a minimum of one of the roots will be on the imaginary axis.

Suppose $s_y=iw_i, w_i\in\ R, w_r\neq 0$. Then $S_r(s,d)=0$ clears that both of its poles, i.e. real and imaginary are zero and given by the following equation.

$-w_i^2-a+\phi_i sin(d_iw_i)-\varphi_i cos(d_iw_i)=0$ and

$-bw_i+\varphi_i sin(d_iw_i)+\phi_i cos(d_iw_i)=0$

Rearranging above equations, we may write as

$$\phi_i sin(d_iw_i)-\varphi_i cos(d_iw_i)=w_i^2+a \qquad (25)$$

$$\varphi_i sin(d_iw_i)+\phi_i cos(d_iw_i)=bw_i \qquad (26)$$

Adding eq. 18 and 19, we get

$$sin(d_iw_i)[\phi_i+\varphi_i]+cos(d_iw_i)[\phi_i-\varphi_i]=w_i^2+a+bw_i \qquad (27)$$

Now, further substituting value in Eq. 27, we get following values

$$sin(\,d_iw_i)=[(k_1^2+k_2^2w_i^2)|\lambda_i|^2]^{-1}[\phi_i(w_i^2+a)+\varphi_i bw_i] \qquad (28)$$

$$cos(\,d_iw_i)=[(k_1^2+k_2^2w_i^2)|\lambda_i|^2]^{-1}[\phi_i bw_i-\varphi_i(w_i^2+a)] \qquad (29)$$

Taking square of Eqs. 28 and 29 and adding them gives us Eq. 30.

$$(w_i^2+a)^2+b^2w_i^2-(k_1^2+k_2^2w_i^2)|\lambda_i|^2=0 \qquad (30)$$

From Eqs. 29 and 30, we may also derive $tan(\,d_iw_i)=\psi_i$and $d_i=k\pi+tan^{-1}(\psi i)w_i^{-1}$, where k is the smallest integer such that $d_i>0$. Further two cases are given below

3.2.1.2 Case 1 – When the imaginary part is zero, i.e. $Im(|\lambda_i|)=0$: In this case, we have only real values. From Eq. 30, it is cleared that it has two real roots $w_{i1}>0$ and $w_{i2}=-w_{i1}$. The negative root is neglected as we have considered only the positive part and also imaginary roots for $S_i(s,d)$ forms conjugate pairs. If we have fix value i.e $\lambda_i>0$ then we may write as $\Psi_{i1=-}\Psi_{i2}$. Therefore, $w_{i_1}{}^{-1}tan^{-1}\psi i1=w_{i_2}{}^{-1}tan^{-1}\psi i2$. From this, we can derive easily $d_{i1}=d_{i2}$. We concluded that if the multi-agent system achieves consensus for d = 0, then the roots of the equation will be in the LHP for all $d_i\in[0,d_i{}')$. As roots of the equation move away from the centre to the negative side (i.e. negative eigenvalues, the system becomes more stable). Therefore, a stable system must have roots in the LHF. The presence of roots in the RHP suggests that the system has a positive eigenvalue that causes the system to become unstable. For a system to become gradually stable, it should have at least one complex root.

3.2.1.3 Case 2 – When the imaginary part is not zero (i.e. $Im(|\lambda_i|)\neq 0$): We have to consider both real and imaginary parts i.e $Re(|\lambda_i|)+Im(|\lambda_i|)$. Let $s_i=i.w\_i, w\_i\neq 0$ and $i\in[2,3,4..n]$ are the imaginary roots then we have Eq. 31.

$$(w_i^2+a)^2+b^2w_i^2-(k_1^2+k_2^2w_i^2)|\lambda_i|^2=0 \qquad (31)$$

Let the root of the above equation are $w\_i_1>0$ and $w\_i_2=-w\_i_1$; then, we may write as $w_{i1}=w\_i_1$ and $w_{i2}=w\_i_2$. Therefore, $\phi_{i_1}=-\phi_{i2}$, $\varphi_{i_1}=\varphi_{i2}$, $\phi_{i_2}=-\phi_{i1}$, $\phi\_i_2=\phi_{i1}$. From this, we can easily derive $\Psi\_i_1=-\Psi_{i_2}, \Psi\_i_2=-\Psi_{i_1}$ and $tan^{-1}(d\_{i_1}w\_i_1)=-tan^{-1}(d\_i_2w\_i_2)$. Thus, we can simply write $d\_i_{1=}d_{i2}$ and $d\_i_{2=}d_{i1}$. We concluded that in the case of real and imaginary roots, stability function (i.e $S(s,e^{-ds})$) contain poles on an imaginary axis such that $0<d'<$ d and $S(s,e^{-d_0s})$ becomes gradually stable for all $d^0<d'$. Moreover, complex roots responses tend to decay with time in the LHF and complex roots responses tend to increase with the time in RHF. Therefore, for a system to be stable, complex roots should be in the LHP, and time delay d_i will be used for $w\_i>0$.

4. Simulation and Results

To verify the effectiveness of the proposed method, we consider a second-order network of vessels borrowed from [28], which follows a directed topology for communication. Therefore, a minimum delay margin '*dr*' between the interacting agents can be calculated. The system model can be represented by equations, $x'(t) = v_i(t)$ and $v_i'(t) = 2x_i(t) + v_i(t) + u_i(t)$.

Taking Laplace of the above equations to interpret in the frequency domain, we have $SX_i(s) = V_i(s)$ and $SV_i(s) = 2X_i(s) + V_i(s) + U(s)$. And, to find the transfer function, we substitute the value of V_i(s), we get $S^2X_i(s) = 2X_i(s) + SX_i(s) + U(s)$ and $\frac{X_i(s)}{U(s)} = \frac{1}{(S^2+S-2)}$.

For experimentation, the proposed method is applied on an undirected graph. We introduced a communication delay problem between the interacting agents. Here the first agent is directly accessible to external input u(t). And all the eigenvalues are non-zero and lies in the LHP. According to stability condition, for any value of delay (d), system dynamics becomes stable gradually if and only if roots of Eq. 30 or eigenvalues $\lambda_2 = 1, \lambda_3 = 2, \lambda_4 = 4.5 + \frac{\sqrt{23}}{2}i, \lambda_5 = 4.5 - \frac{\sqrt{23}}{2}i$ belongs to the complex numbers. Now, if we have k_1= -0.8+11i and k_2 = -0.8-11i, from theorem 2, the system achieves consensus and does not affect the stability of the system in a delay free case as shown in Fig. 4. Communication delay between the agents is calculated d_2 = 0.9, d_3 = 1, d_4 = 1.6, d_5 = 1.7. According to theorem 2, the minimum value of delay from all delay values, i.e. $d_i' = min$ (all the delay values overall root values) is selected. Then, second-order networked multi-agent systems exhibit consensus. Delay free and with delay, cases are shown in Figs. 3 and 4. From Figs. 3 – 6, it is cleared that we achieve Consensus in NMAS without disturbing the stability of the system according to consensus conditions $S(s,e^{-ds})$. We have used a minimum of all the delay values i.e. 0.9 between agents to achieve robust consensus without disturbing the stability of the system. As stated earlier, the stability function (i.e. $S(s,e^{-ds})$) contains poles such that $0 < d' <$ d and $S(s, e^{-d_0 s})$ becomes gradually stable for all $d^0 < d'$. Moreover, the response of agents tends to decay with time to a single point. This shows that all the agents in the MAS have reached a single point agreement i.e achieved consensus. Therefore, for a system to be stable and achieve consensus, time delay d should be $0 < d' <$ d. Figs. 5 and 6 show the result of system when delay is 1 and 1.2, simultaneous, according to stability function $S(s, e^{-d_0 s})$.

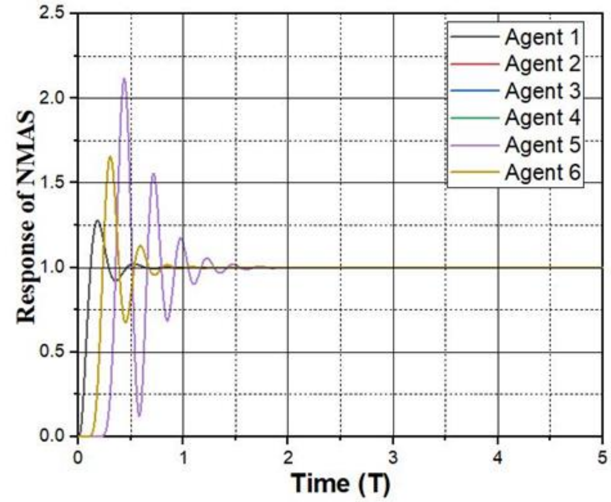

Fig. 3: Convergent response when delay = 0.9

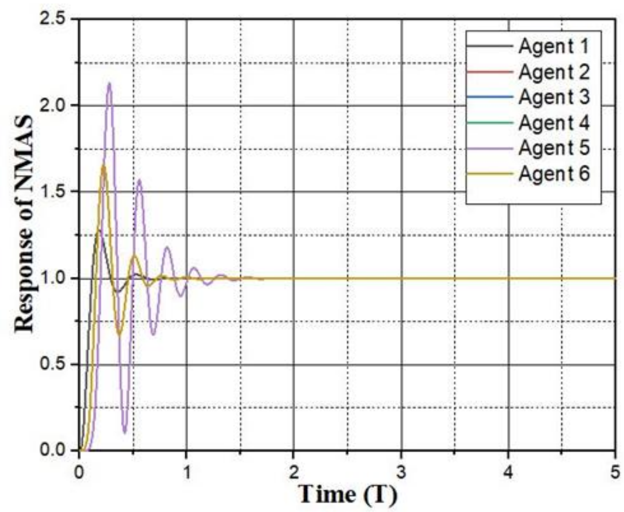

Fig. 4: Convergent response when delay = 0

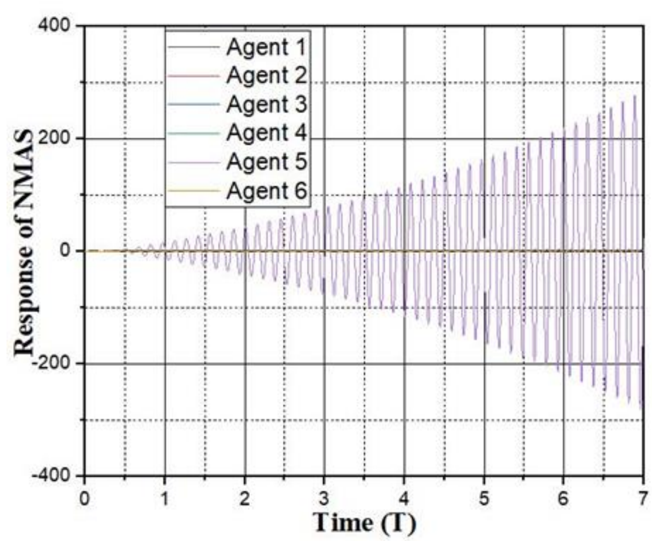

Fig. 5: Divergent response when delay = 1

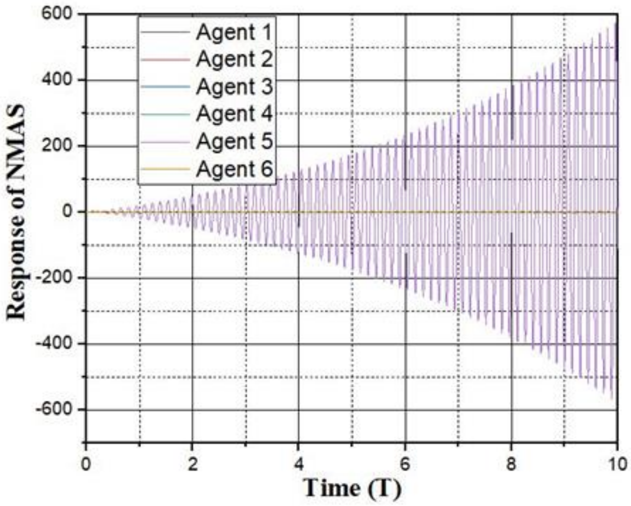

Fig. 6: Divergent response when delay = 1.2

Further, Figs. 3–6, show the response of coordinating agents in NMAS. Assume that there is no delay then the response of the agents reaches to the single consensus point although there are spikes in the beginning as in Fig. 3, which is called as a convergent response. Similarly, Fig. 4 shows the response of the agents in the presence of a minimum delay 0.9. According to the stability condition as in theorem 2, agents reach the single consensus value despite minimal fluctuations in response. Similarly, Fig. 5 and Fig. 6 shows the divergent response of the NMAS when the delay value is increased from 0.9 to delay = 1 and 1.2. All the coordinating agents do not reach the single consensus value, but response fluctuates with the time called divergent response.

Furthermore, if we have real and imaginary roots, stability function (i.e $S(s,e^{-ds})$) should contain poles on an imaginary axis such that $0 < d' <$ d and $S\left(s, e^{-d_0 s}\right)$to make the system stable for all $d^0 < d'$. Moreover, the response of the system remains convergent when the delay margin is known by the agents and the delay margin should be the minimum of all delay values between agents. Therefore, if the above conditions satisfy then MAS show a convergent response and achieves a consensus when all the agents agree on a single point. Furthermore, sensitivity function $S_n(s)$ quantifies how sensitive $F_c(s)$ is to the variation in the system transfer function i.e., $F(s)$. Here objective is to check the consensus performance of the NMAS. In order to quantify the performance, an index of 'smallness' of the error is used. In other words, the error of the NMAS should be as small as possible to be called good performance. This means system with less error more sensitive and large error less sensitive. To achieve this, the proposed controller is designed with the help of $S_n(s)$ and $F_c(s)$. The closed-loop response of NMAS is shown in Figs. 3 and 4 under the variation of time delay. These results show that before 1.5 seconds, there occur oscillatory errors in the system because of the existence of time delay. But these variations vanish after 1.5 seconds and steady state error minimized with the proposed controller as compared with [29]. Hence, simulation results are consistent with the theorem.

5. Conclusion

This research described the performance of consensus control of second-order networked multi agent systems with induced delay. We proposed a new model to design an analytical *H2* controller for performance index using the controller parameterization technique. We discussed the delay margin to ensure consensus for the second-order NMAS with and without communication delay. By using delay free conditions, we devised a clear formula to achieve consensus for time delay second order systems. Then, we verify our methods with the help of numerical examples. This model has some problems with higher-order system. Therefore, in the future, we will work on higher order systems.

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