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Numerical analysis for mechanical behaviour of beams made by composite and functionally graded materials

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ABSTRACT

Functionally graded materials are important in laboratory and industry. These materials have opened a new area of research. These functionally graded materials can be produced as a result of composite material by the adding more materials to get desired properties to make it perfect for suitable applications. In this paper, the focus is on the modelling of functionally graded material that is developed to resist wear by any friction or erosion by any liquid. Cemented Carbide (WC) was selected as a composite material that is commonly used in industrial or lab. Before developing the model of functionally graded material (FGM), a detailed study has been carried out on the composite material using the finite element method to find a numerical solution and compared it to the exact one. The analytical solution was obtained by using MATLAB and numerical solution by ANSYS. Besides these software's SolidWorks and SkyCIV programs were utilized for analysis and plotting purposes. At the end of the project, one layer titanium carbonitride (TiCN) was added to one face and the same layer was added to the four faces to make the comparison between the two cases to select the best model using the results of a total of deformation.

1. Introduction

Functionally graded materials (FGMs) are novel materials having dimensions related properties gradually changeable with time [1]. It is the advanced development technique of formerly used composite materials [2]. FGMs are also a kind of composites, having two or more materials prepared by continues changing of material composition [3]. It is consisting of two or more materials to achieve the desired properties according to the application where an FGM is used. FGMs have obtained a great attention of researchers in the past decade due to their graded properties at every single point in various dimensions [4-5]. These material properties can be enhanced by optimization through numerical method or simulation by finite element analysis [6]. Modelling and simulation of functionally gradient materials is important for engineers to be accurately predict their mechanical behaviours and then utilize at for proper application like beams etc. [7]. The characteristics of functionally graded cemented tungsten carbide are the best example for the performance of material as a graduate to surface. Therefore, the material has a significant combination of wear resistance and fracture toughness [8]. The functionally graded materials have high resistance to the heat from one face or the one side that makes it that of high strength and good thermal conductor. These kinds of studies can open a new horizon to produce different new products as per desired characteristics [9]. The main role of applying functionally graded materials to cemented carbide tool materials is to enhance and develop a good tool material that leads to high wear resistance [10]. Titanium carbonitride (TiCN) contains a kind of grains that give a high-density structure as a result and these properties give finally the material high hardness and toughness [11]. The TiCN can be prepared by reduction of titanium oxide with boron carbide and carbon, purified under high temperature and vacuum to obtain small size particles [12]. It was described in detail the process or method for how to manufacture the functionally graded cemented tungsten carbide with a surface that will have high wear-resistant and toughness. This surface is optimized by a layer fabricated through a carburization and heat treatment process and followed by a conventional liquid phase sintering technique [13]. All materials that have traditional properties such as materials or composite material will become a failure when faced with extreme mechanical and thermal loading [14]. But the functionally graded material can enhance characteristics of materials such as Poisson's ratio, shear modulus, Young's modulus, and material density [15]. Kernel particle method (RKPM) was used for elastic mechanical domains of the functionally graded materials (FGM). The control parameter of influence by the domain radius, penalty factor and different node of distribution based on calculation accuracy are studied [16]. Functionally graded material are utilized for biomedical applications especially in orthopaedic purposes [17].

Birsan et al. [18] worked on composite beam of made of functionally graded materials and foam. Theoretically and experimentally there was good agreement this was main conclusion after studying deformation of composite beams. Assem et al. [19] worked on functionally graded materials using finite element method technique. They mainly focused on modal analysis by obtaining natural frequencies of functionally graded material beams. Similarly Yilmaz et al. [20] worked on free vibration theoretical and numerical study of functionally graded made material beams. This numerical method used was again finite element based and finally they calculated elasticity of beams as result. In this work the focus was to attenuate the vibration amplitude in CESNA 172 airplane wing by using Functionally Graded Material instead of uniform or composite material. Wing strength was achieved by means of stress analysis study, while wing vibration amplitudes and shapes were achieved by means of Modal and Harmonic analysis [21]. Liu et al. [22] worked on functionally graded materials to study elasticity by using reproducing kernel particle method. There results concludes that this method is effective and nice approach for study. Alasti et al. [23] worked on the study of functionally graded made cantilever micro-beam subjected to electrostatic pressure and temperature. The ceramic material taken was composed of 0 to 100% from top to bottom surface. This student focuses on the effects of temperature changes and the non-linear electrostatic pressure on the deflection and stability. Also normal stress distributions on the cross section of the beam under the mentioned loading is studied. Eremeyev et al. [24] proposed a new beam model that consists of a novel kind of shape function for the study of shear stress distribution and deformation in the transverse loading conditions. They concluded that the defect occurs in the FGM beam due to the boundary conditions used with applying lower degree of freedom (DOF) [24].

Engineering structures are so designed and constructed that stability, strength, Buckling, stiffness, and durability are ensured during their lifetime for all anticipated loads [25]. Among these, the most usually used structural components are the beams which are having multitudinous engineering-oriented practical applications. They undergo both static as well as dynamic loads of elements. There is a necessity to construct a structure to work securely along with its tenure of the service period. In this paper, the total deformation for cemented carbide for four different cases are studied.

1. Cantilever beam (fixed-free) with end load.
2. Simply supported beam with a centre load.
3. Simply supported beam with moment load applied at the middle plane.
4. Fixed-fixed beam with a centre load.

The static manually (exact solution) obtained by MATLAB Program and doing the same study at 4 points by finite element method using (ANSYS program) (approximate solution) and (SolidWorks Program) to draw the modelling. After that, cemented carbide will be developed to become a functionally graded material. Also, analysis is carried out based finite element method to make the treated material. In beginning, there is no information about the mechanism of functionally graded material. After studying the literature, the mechanism is understanding and implemented practically in this paper for functionally graded material. The study of functionally graded materials was a challenge to study in the ANSYS program. After detailed analysis by simulation, it is concluded that to select material layers by a layer that characteristics can be improved of an element. After studying different materials and their properties it is decided to select TiCN for the addition of layer because TiCN has good toughness, adhesion, and resistance to chipping. It was a great challenge of how can make a merging surface between the layer of cemented carbide to make it a functionally graded material (i.e. TiCN) as one element. Also, the suitable boundary condition at some specific thermal conditions to make a comparison of total deformation and maximum principal stress was a challenge.

2. Methodology

2.1 Numerical Method

In the numerical method, simulation in the ANSYS program is required to do static structural analysis for the shown cantilever beam. The beam is made of material of Cemented Carbide. The dimensions are shown in Fig 1.

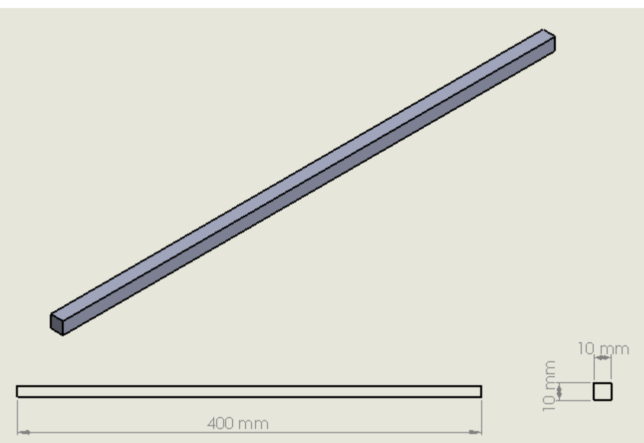


Fig. 1. Geometry and dimensions of cemented carbide by SolidWorks program

The objective here was to compare the transverse deflection of the beam using the finite element with the exact solutions that can be calculated from analytical equations. For the sake of the comparison purpose, following four cases were considered.

1. Cantilever beam (fixed-free) with end load
2. Simply supported beam with a centre load
3. Simply supported beam with moment load applied at the middle plane
4. Fixed-fixed beam with a centre load

2.2. Analytical Method

2.2.1 Cantilever beam (fixed-free) with end load

The cantilever beam (fixed-free) with end load was measured as shown in Fig. 2 by using Eq. 1.

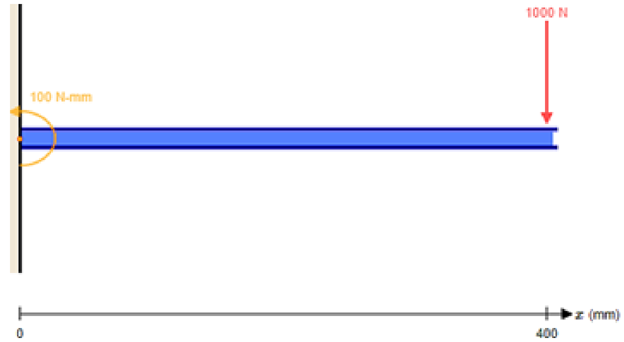


Fig. 2. Cantilever beam (fixed-free) with end load

$$Y_{max} = -\left(\frac{F\,L^3}{3EI}\right) \tag{1}$$

2.2.2 Simply supported beam with a bending load at centre

The simply supported beam with a bending load at centre was measured as shown in Fig. 3 by using Eq. 2.

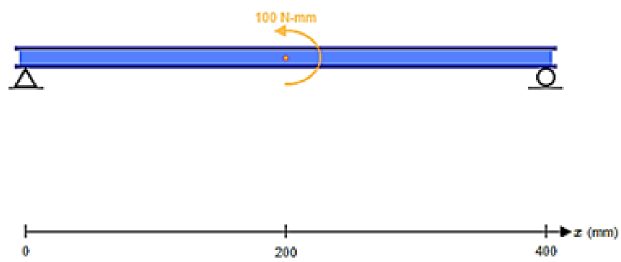


Fig. 3. Simply supported beam with a bending load at centre

$$Y_{max} = \frac{Fa\,(L-x)}{6EIL}(x^2 + b^2 + 2Lx) \tag{2}$$

2.2.3 Simply supported beam with moment load applied at the middle plane

The simply supported beam with moment load applied at the middle plane was measured as shown in Fig. 4 by using Eq. 3.

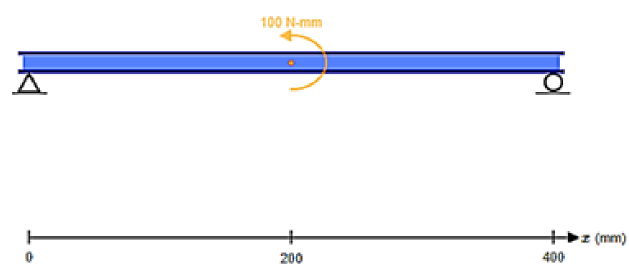


Fig. 4. Simply supported beam with moment load applied at the middle plane

$$Y_{max} = \frac{M_b X}{6EIL}(x^2 + 3a^2 - 6L + 2\,L^2)) \tag{3}$$

2.2.4 Fixed-fixed beam with a centre load

The Fixed-fixed beam with a centre load was measured as shown in Fig. 5 by using Eq. 4.

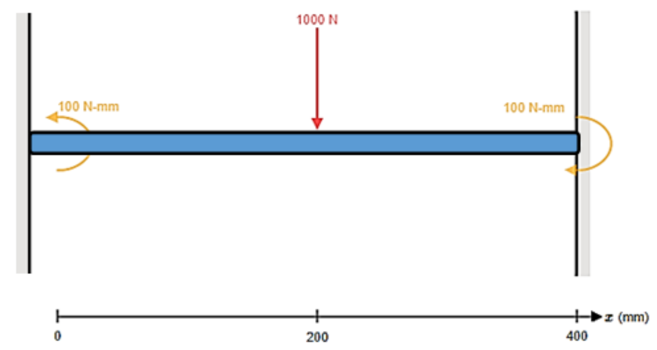


Fig. 5. Fixed-fixed beam with a centre load

$$Y_{max} = \frac{Fx^2}{48EI}(4x - 3L) \quad (4)$$

For Eqs. 1 – 4, L is the length of the beam, F is the applied load, E is elastic modulus, I is moment of inertia, and M is bending moment.

3. Results and Discussion

3.1 Validation of Model

For validation of the ANSYS model, the results of four different cases were compared with the analytical solution obtained from analytical formulas and tried by MATLAB code for the solution.

3.1.1 Case 1 – Cantilever beam analysis

In the case-1, cantilever beam of cemented carbide material was studied by numerical simulation in ANSYS by finding the total deformation. The boundary condition used was as one end was fixed and an anticlockwise moment of 100 N.mm was applied at the other end with the help of 1000 N applied load. The results in Fig. 6 show that the magnitude of maximum total deformation is 46.518 mm as an approximation solution.

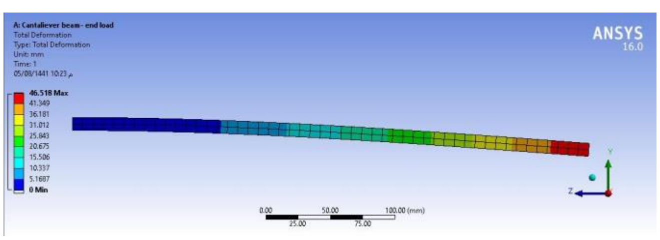


Fig. 6. Total deformation of the cemented carbide cantilever beam

For the direction of total deformation to compare with MATLAB result, result given in Fig. 7 is obtained. For getting analytical solution MATLAB is used and the result obtained is represented in Fig. 8.

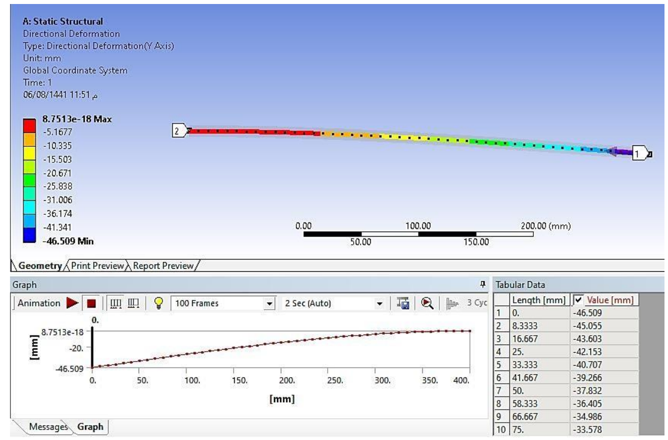


Fig. 7. Total deformation of -46.509 mm (max.) of cemented carbide, simply supported beam with load at the midpoint of the span

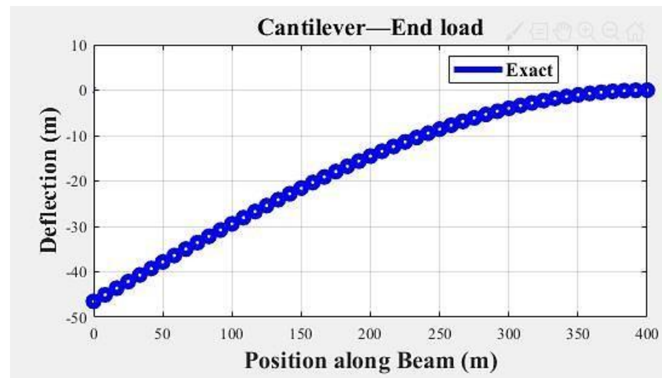


Fig. 8. MATLAB graph for calculation of total deformation for cantilever beam

Table 1 shows that the results of numerical and analytical solutions are very close.

Table 1

Comparison between approximate and exact solution for maximum total deformation

Comparison	Total deformation
ANSYS result	-46.50 mm
MATLAB result	-46.51 mm

3.1.2 Case 2 – Simply supported beam with a centre load analysis.

In this case, the cantilever beam of cemented carbide by applying load at the centre or midpoint was analysed in the ANSYS program to find the total deformation. The boundary condition used in simply supports pin support was applied at one end and a load of 1000 N was applied at the centre of a beam.

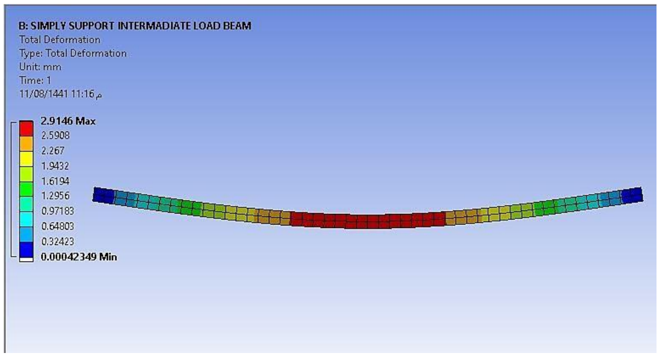


Fig. 9. Total deformation of 2.9 mm (max.) of cemented carbide, simply supported beam with load at the midpoint of the span

As given in Fig. 9, after solving the maximum and minimum magnitudes, values of total deformation found are respectively given as 2.9146 and 0.00042349 mm. In the next step, we obtained the next direction deformation for comparison purposes with MATLAB program results.

By using Eq. 2 with the same inputs in the MATLAB program resulted as Fig. 10. The x-axis represents the position along beam or length and the y-axis represents the deflection or total deformation which is -3 mm very close to the numerical result of -4.2314 x 10^{-5} mm.

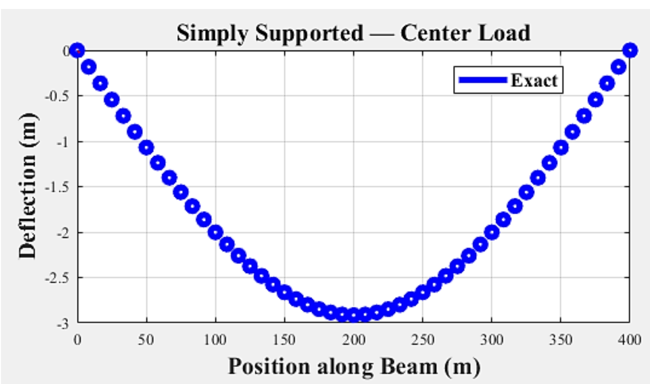


Fig. 10. Direction of total deformation (-3×10^{-5} mm) of cemented carbide, simply supported beam with centre load of 1000 N by MATLAB

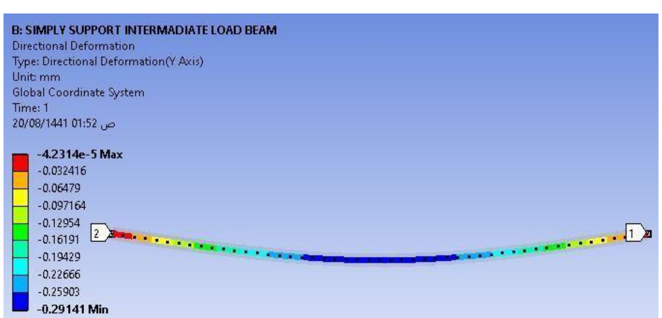


Fig. 11. Direction deformation (-4.2314e^{-5} mm Max) of cemented carbide, simply supported beam under centre or midpoint load of 1000N

Table 2 shows that the numerical and analytical solutions are not same.

Table 2

Comparison of results for the simply supported beam with central loading

Comparison	Total deformation
ANSYS result	-4.2314×10^{-5} mm
MATLAB result	-3×10^{-5} mm

3.1.3 Case 3 – Simply supported beam with a centre moment analysis

In this case, simply supported beam of cemented carbide with moment load was directly applied at the centre to find the total deformation ANSYS program. The boundary condition simply supported pin support from the left hand and roller support from the right hand and applies moment load in the centre of a beam by 100 N.mm. The results in Fig. 12 show that the magnitude of maximum and minimum total deformation values are respectively given by 0.00028108 and 2.2098 x 10^{-7} mm.

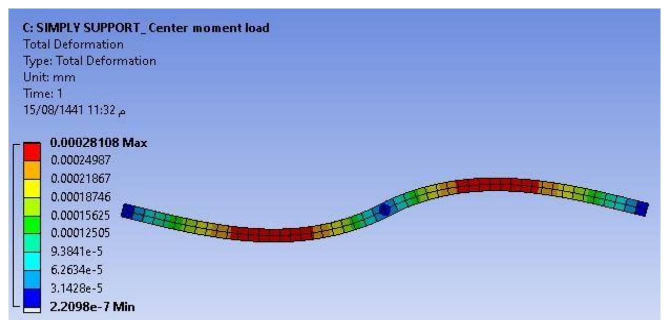


Fig. 12. Total deformation (0.00028 Max) of cemented carbide, simply supported centre moment of 100 N.mm

The direction deformation to make a comparison with MATLAB program resulted as Fig. 13.

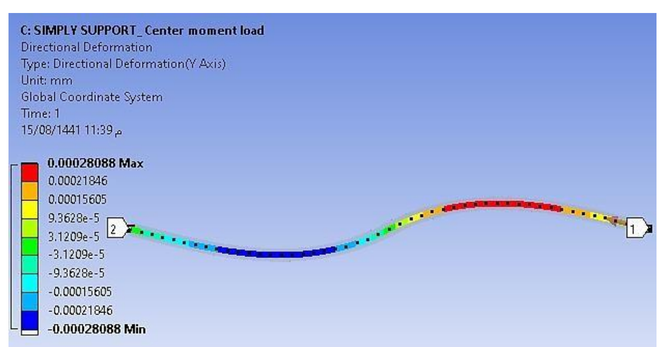


Fig. 13. Direction deformation (0.00028088 mm) of cemented carbide, simply supported centre moment load of 100 N.mm

The Eq. 3 using the same input of boundary condition and moment at centre resulted as Fig. 14.

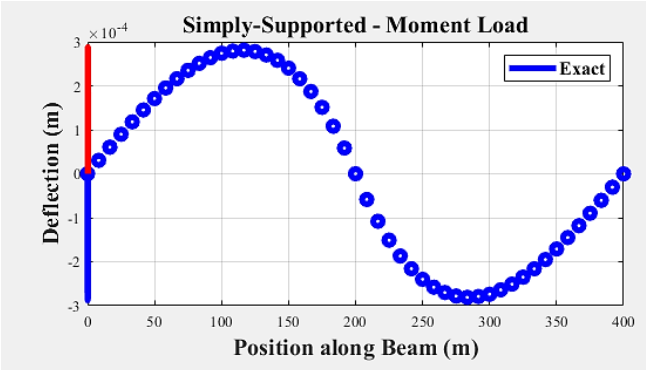


Fig. 14. Direction of total deformation (0.000299 mm) of Cemented Carbide simply supported beam with centre load of 100 N.mm by MATLAB

Table 3 shows that the results of numerical and analytical solutions are very close.

Table 3

Comparison of results for the simply supported beam with a central moment

Comparison	Total deformation
ANSYS result	-2.80×10^{-4} mm
MATLAB result	-2.81×10^{-4} mm

3.2 Functional Graded Material Analysis

The validated model is utilized in this section for analysis of function graded materials. First, the focus was on the study of cemented carbide as one layer at thermal condition before developing to a fully functionally graded material. After applying the boundary condition at fixed support at the thermal condition of 1000 °C, the maximum total deformation obtained is equal to 0.20741 mm, as given in Fig. 15.

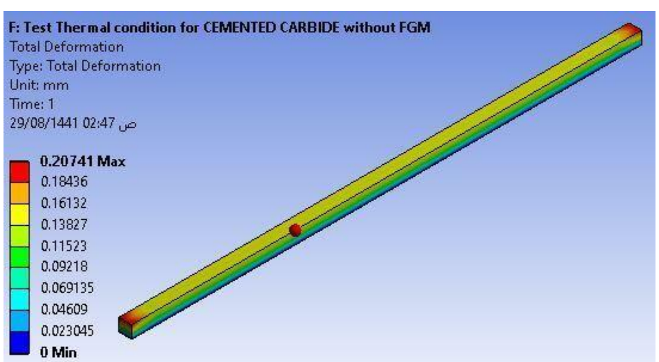


Fig. 15. Total deformation at the thermal condition of 1000 °C. The maximum total deformation produced of 0.2 mm

As shown in Fig. 16, in the next step, a layer (1.5 mm) of TiCN on one face for cemented carbide was added which resulted in deformation reduction because the temperature was absorbed. A boundary condition of temperature 1000 ºC on a surface of TiCN was applied to find the total deformation and maximum principal stress on TiCN and cemented carbide.

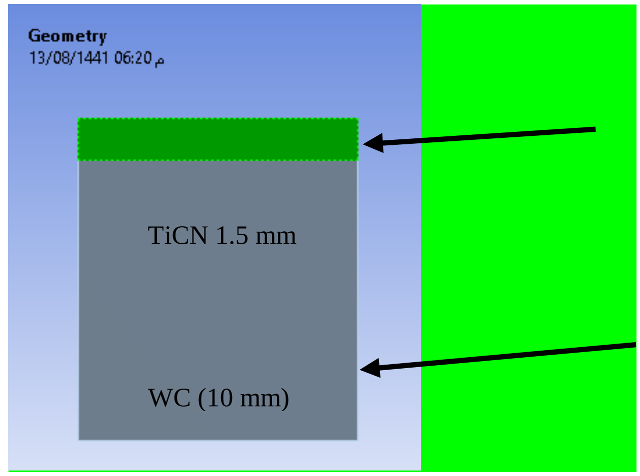


Fig. 16. Adding layer of TiCN, 1.5 mm in cemented carbide

As given in Fig. 17, after applying boundary conditions at the fine mesh and fixed support from the bottom of the beam with the thermal condition at a temperature of 1000 °C, the maximum total deformation found was 0.10081 mm. The new layer of TiCN absorbed the energy to decrease the wear resistance and protected the cemented carbide from power or load that coming from outside.

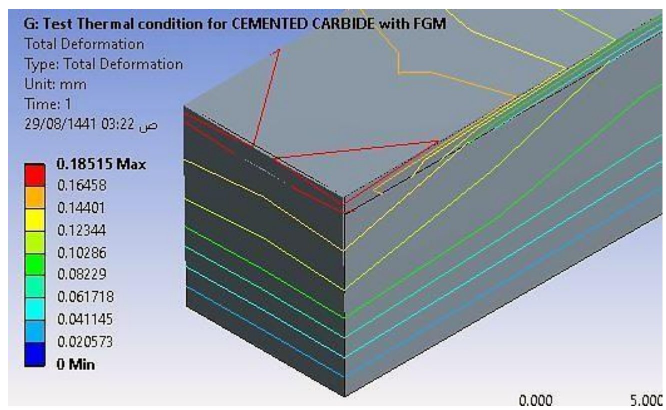


Fig. 17. Total deformation (0.18515 mm) at 1000 °C thermal condition

Thus, it is needed to develop and enhance this layer to find the best value of total deformation. The Figs. 18-21 show the maximum deformation that occurred on the surface of TiCN, as observed it was minimum in the bottom of cemented carbide. The difference in total deformation can be seen when compared to the study of cemented carbide without layer before developing the material to be functionally graded. Then, normal force of 250 N was applied on TiCN without thermal condition with fixed support in base cemented carbide to check the total deformation and Maximum principal stress.

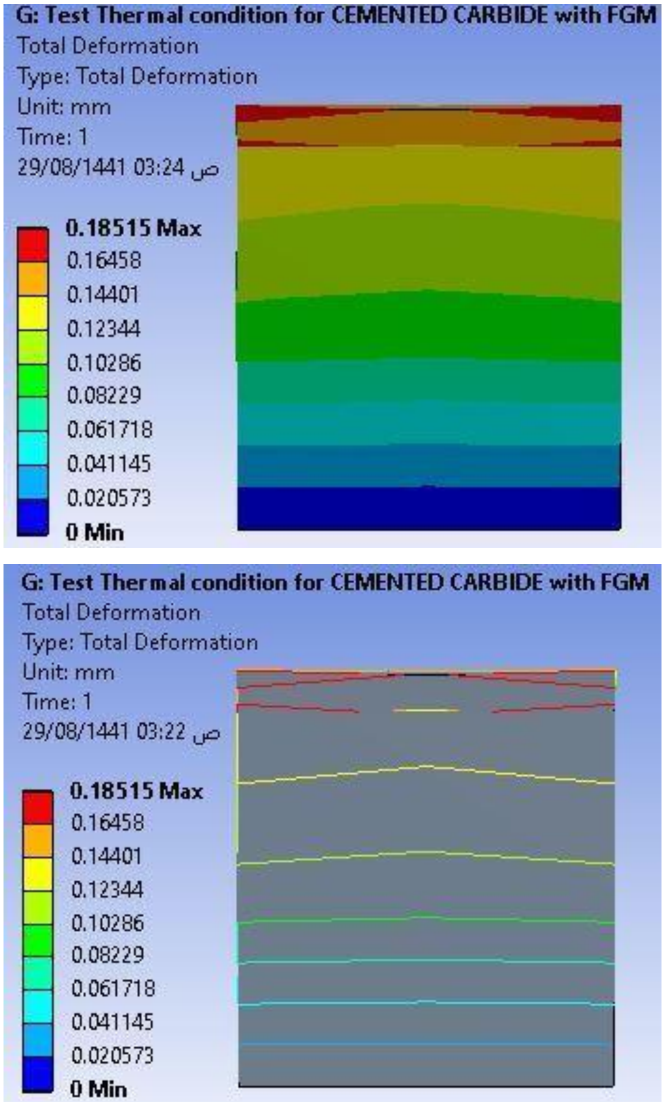


Fig. 18. Graph of total deformation Wireframe shape

The cemented carbide is mostly used as a tool drill in the factory. Therefore, there is a need to check the deformation under loading conditions by considering with and without thermal condition cases. Thus, as shown in Fig. 19, applying boundary conditions at fixed support from the bottom of the beam without thermal condition and apply normal load on TiCN directly, the maximum total deformation obtained was equal to 3.3041 x 10^{-6} mm. In the next step, a normal load of 250 N on TiCN with the thermal condition and fixed support was base cemented carbide to check the total deformation and Maximum principal stress.

In Fig. 20, the total deformation and maximum principal stress for two layers at 500 °C and apply force normal force of 250 N on TiCN with fixed support in base cemented carbide. Now applying the boundary condition at fixed support from the bottom of the beam with the thermal condition of temperature 500 °C under normal load on TiCN directly. The maximum principal stress and maximum total deformation obtained are equal to 0.033461 MPa and 0.3747 mm respectively.

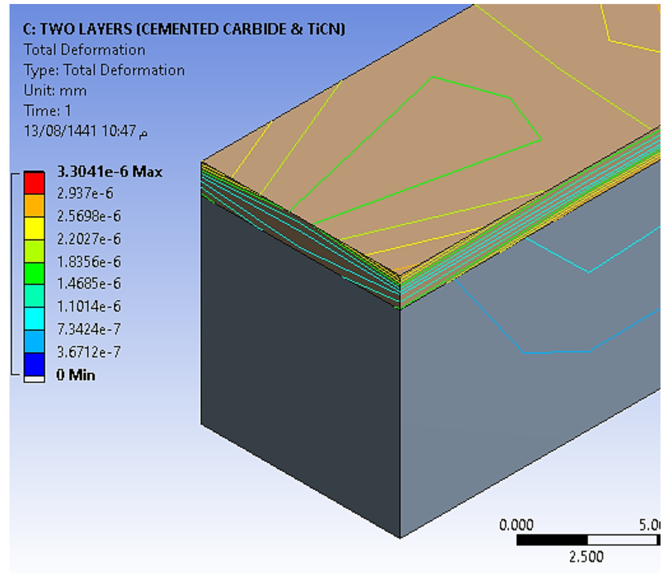


Fig. 19. Graph of total deformation normal force on the TiCN surface, 250 N load without thermal condition

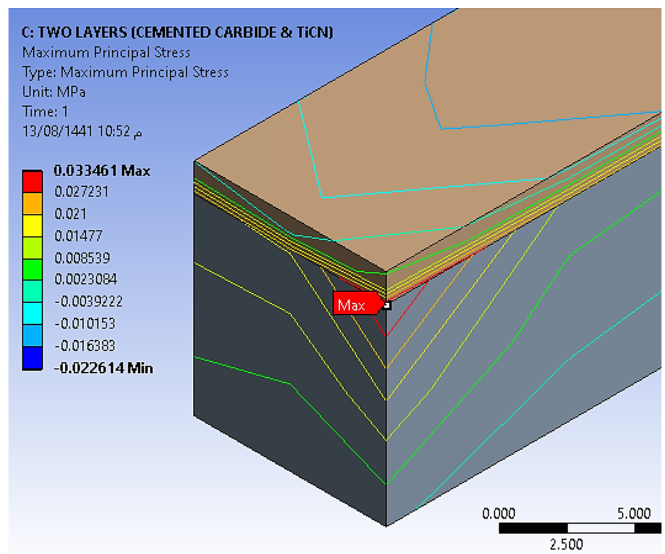


Fig. 20. Maximum principal stress normal force on the TiCN surface without thermal condition

From above all studies done until now, the material needed to be more enhanced and also there was a need to reach the best conditions for functionally graded material. Threrefore, in the next step, iterative method was to be used for concluding more details about the thickness of the TiCN layer. The specific boundary conditions used were fixed support from the bottom of cemented carbide and applied a normal load of 250 N on face the layer with the thermal condition of a temperature of 500 ºC for the whole body. This iterative approach aimed to find the best total deformation at the best TiCN layer thickness. Let’s suppose, starting from 1 mm from one face and applying all loads on it directly.

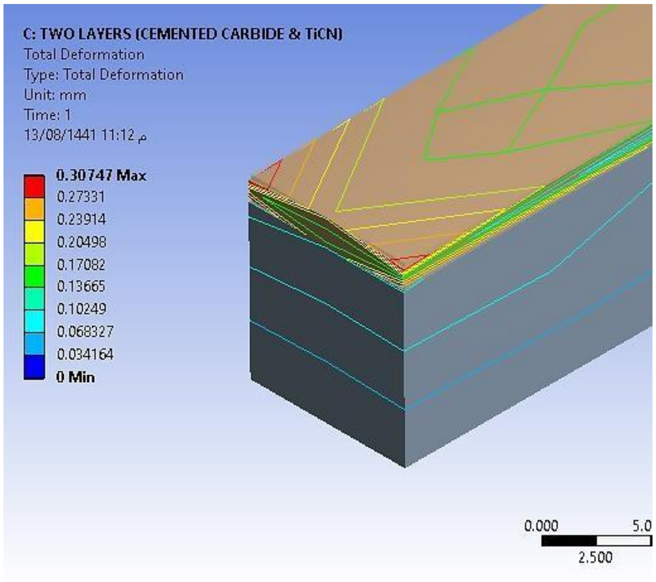


Fig. 21. Graph of total deformation normal force on the TiCN surface, 250 N load with thermal condition of 500 °C

When a normal load of 250 N was applied at a thermal condition of temperature 500 °C on the whole elements it was observed that if the thickness is increased, and the total deformation decreases.

Table 4

Change total deformation according to changing the thickness of Titanium Carbonitride

Thickness of layer of TiCN (mm)	1	1.5	2	2.5
Total deformation (mm)	0.12733	0.12733	0.1256	0.12512
Thermal condition (°C)	500	500	500	500

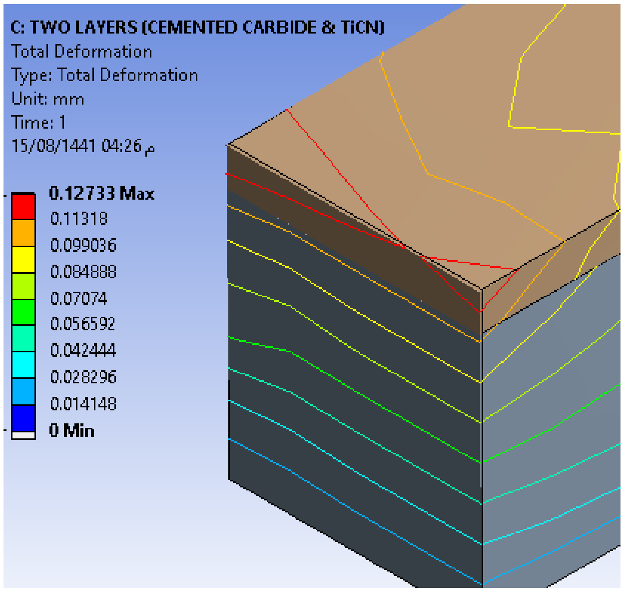
From a previous iterative study, it can be concluded that the total deformation is decreased when the thickness of layer TiCN increases. However, the increase up to three digits enhancing was possible and further increase of thickness had no impact on the results. Therefore, there was need of some other technique that could result in the lightweight of the material and made it more wear resistive with low total deformation. For this purpose, a new iterative method was implemented based on the thickness of the layer for TiCN, however, this time it was one thickness for each face. Then specific boundary conditions were applied for all stages. Such as fixed support from the bottom of cemented carbide and a normal load of 250 N was applied on the face of the layer at a thermal condition of a temperature of 500 °C for the whole body. The purpose of this new iterative method was to find the best total deformation at the best thickness layer of TiCN as shown in Fig. 22.

In Fig. 23, it is noted that cemented carbide was covered by TiCN for each face. As practiced in previous cases, each layer and cemented carbide material properties were defined in ANSYS program and this step was important for developing functionally graded material. The thickness of TiCN was kept equal to 1 mm.

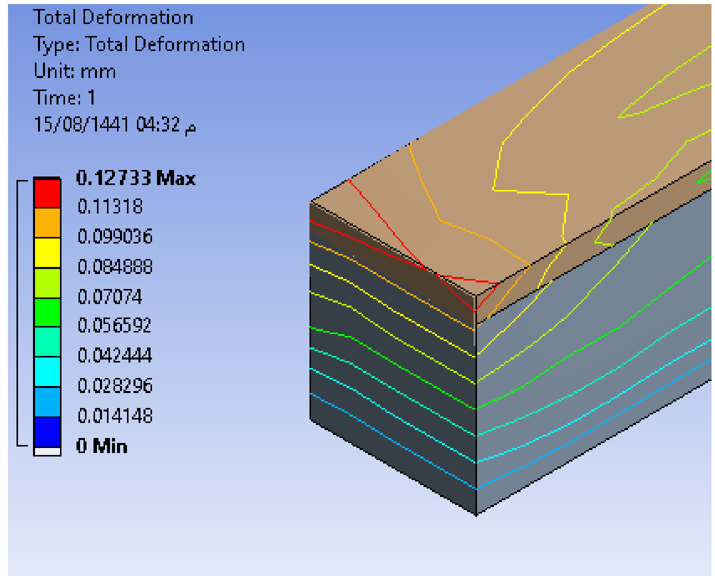
From Fig. 24, the total deformation resulted was equal to 0.10743 mm and it is near to the face at which load was applied. Therefore, after making the comparison between this stage and the previous stage, it is concluded that the solution to make a perfect functionally graded material is possible by selecting the best layer under suitable factors such as load, moment, etc. For this purpose, titanium alloy was added as a layer between TiCN and cemented carbide. The material properties used in the simulation setup were the density of 4.43 x 103 kg/m^3, Yield strength of 880 MPa, tensile strength of 950 MPa, modulus of elasticity 113.8 GPa, shear modulus of 44 GPa, and the poisson ratio of 0.342. The thickness was kept at 0.5 mm and the geometry of cemented carbide finally decreased to an area of 9 mm x 9 mm and length of 400 mm. The layer of TiCN was kept at 1 mm.

After studying and doing analysis from the above discussion, it was observed that the material needed more improvement and enhancement to reach the best value that is required for functionally graded material. To do that again, it was solved by the ANSYS program to discover and conclude more details about the layer and the appropriate sequence of layers was observed, as given in Fig.25. The use of some specific boundary conditions for all stages were required, such as fixed support from the bottom of cemented carbide and apply normal force load 250 N on the face of the layer that changes at thermal condition 500 °C for the whole body. The aim of this exercise was to solve and find the best total deformation at the best appropriate sequence of layers. Therefore, that 0.5 mm for each face was added and the loading was applied directly on the upper face.

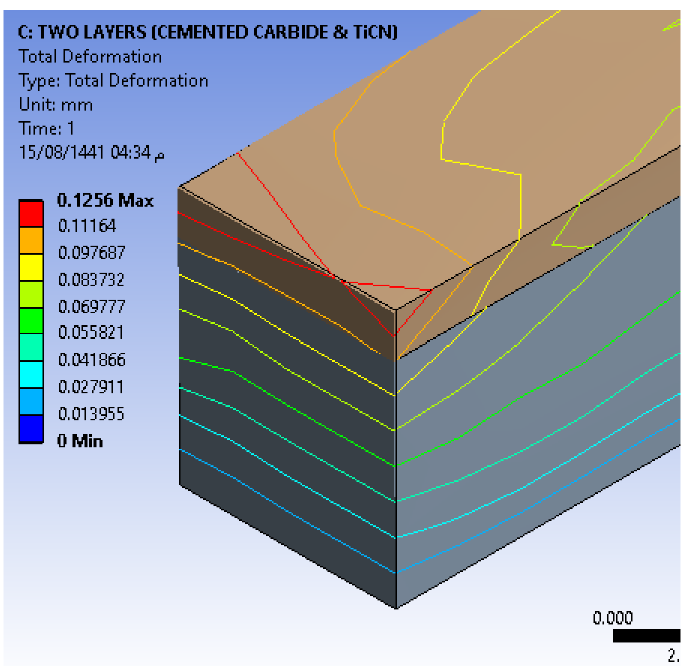
As a result, the total deformation, shown in Fig. 26, was above 0.019424 mm, which means the material reached the best value for achieving the functionally graded material. Therefore, it is concluded that the maximum of total deformation in the upper face reduces gradually in all stages. The reason is that the upper face can absorb the thermal conditions and can control the possible deformation due to environmental conditions.



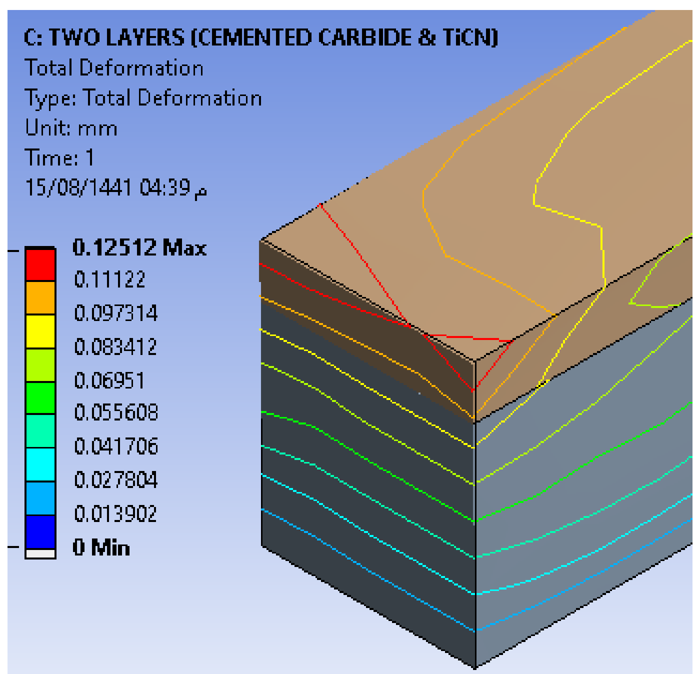
(a) 1 mm of TiCN, total deformation of 0.12733 mm



(b) 1.5 mm of TiCN, total deformation of 0.12733 mm



(c) 2 mm of TiCN, total deformation of 0.1256 mm



(d) 2.5 mm of TiCN, total deformation of 0.12512 mm

Fig. 22. Total deformation of cemented carbide and TiCN beam with thermal condition of 500 °C

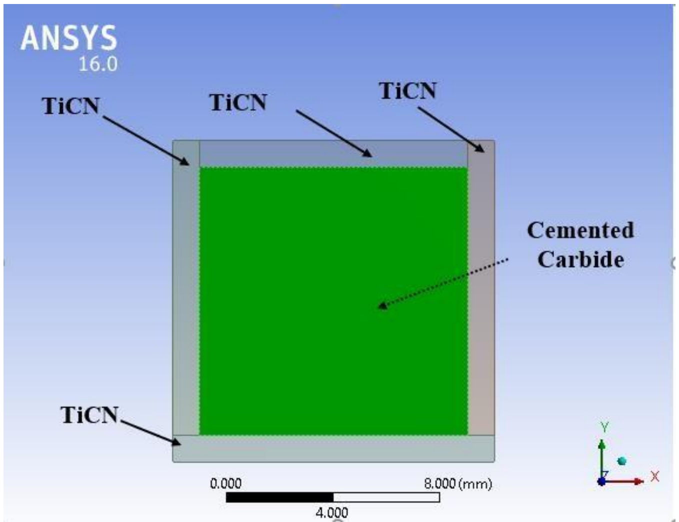


Fig. 23. Modeling of cemented carbide covered all by TiCN thickness of 1 mm

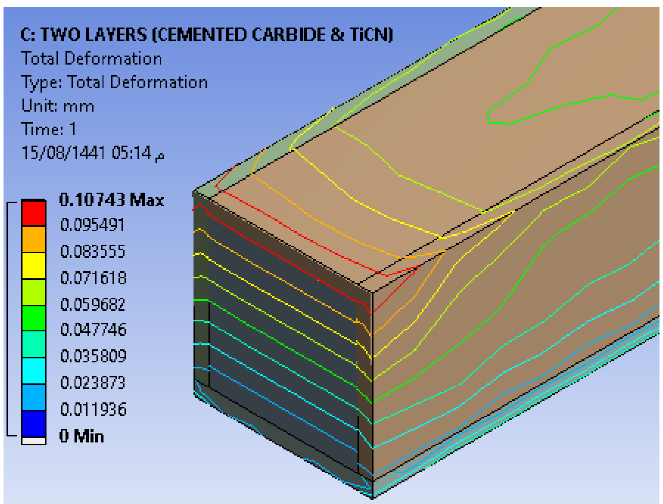


Fig. 24. Graph of total information (0.10743 mm) at the thermal condition for the whole body 500 °C and normal force 250 N

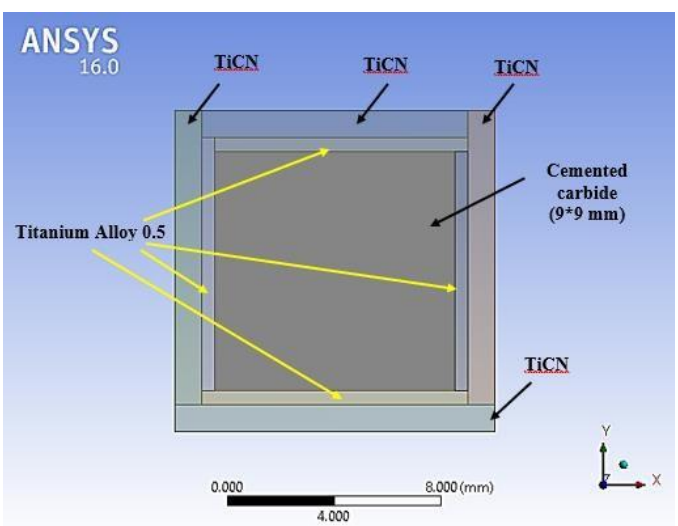


Fig. 25. Three layers cemented carbide, titanium alloy (0.5 mm) and TiCN

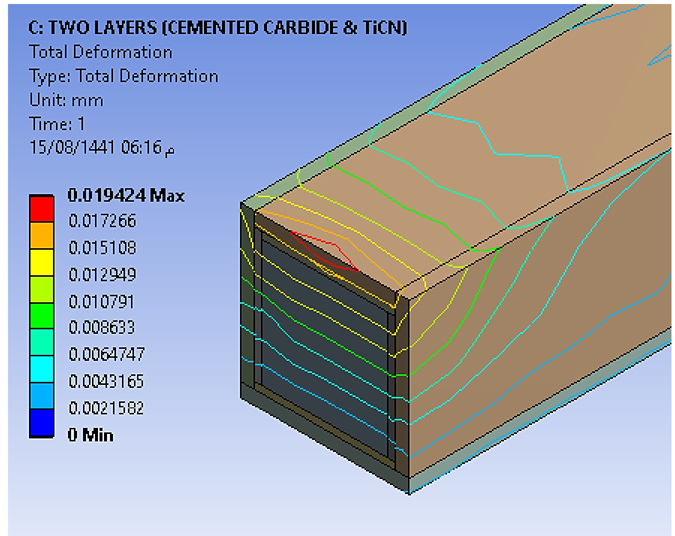


Fig. 26. Total information (0.019424 mm) at the thermal condition for the whole body 500 °C and normal force 250 N

4. Conclusion

Functionally graded materials open a new window for research because of their importance in scientific and practical circles. This study will be useful from the industry point of view. The analysis of this paper focused on finding the critical value for total deformation. This critical value is key for a material to controls its surface to bottom from wear, increase its toughness, improve the gradation of material property such as excellent thermal and consolidate bond strength. All these characteristics can be achieved with the help of total deformation. The best value of maximum total deformation selected based on analysis by iterative numerical approach is 0.019424 mm at a temperature of 500 °C undergoing applying a normal load of 250 N.

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