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Fractional order multi-scheduling parameters based LPV modelling and robust switching H_∞ controllers design for steam dump system of nuclear power plant

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K E Y W O R D S

LPV

Multi-Scheduling

Fractional Order System

Switching Controller

Steam Dump System

Nuclear Power Plant

A B S T R A C T

In this research work, the highly challenging problem of novel modelling and nonlinear control of steam dump system of Pressurized Water Reactor (PWR) type Nuclear Power Plant (NPP) is attempted. The Fractional Order Multi-Scheduling Parameters based Multi-Input Single- Output Linear Parameter Varying (FO-MSP-MISO-LPV) model of Steam Dump System (SDS) is estimated with uncertain dynamics under sudden load variation transients. MSP for uncertain dynamics of SDS in FO framework is the most challenging problem and attempted in a novel fashion for the first time in nuclear industry. Scheduling parameters are dynamic in nature that makes the control problem more challenging. The Model is estimated experimentally by least square method using innovative plant operational data of opening positions of different valves as input variables and steam pressure as an output variable and cold leg coolant temperature coefficient of reactivity, hot leg coolant temperature coefficient, steam flow rate and turbine power as dynamic scheduling parameters. A switching controller is designed to address variable conditions of steam pressure for the actuation of dump valves, relief valves and safety valves in SDS. A robust fractional order LPV switching H_∞ (RFO-LPV-SWH∞) controllers are formulated and designed for FO-MSP-MISO-LPV model. The design of RFO-LPV-SWH$_\infty$ controllers is another significant contribution in switching mode with non-integer and LPV hybrid framework. RFO-LPV-SWH$_\infty$ controllers are tested, simulated and validated against benchmark transients as laid down in Final Safety Analysis Report (FSAR) of PWR-type NPP. The input and output variables at first and second vertex of polytope are fast reference tracking under highly nonlinear uncertain dynamics of SDS. Closed loop simulation experiments are conducted and proved that the proposed closed framework is robust in performance under parametric uncertainty.

1. Introduction

In PWR-type Nuclear Power Plant, conventional PID controller controls the steam pressure under sudden load reduction and grid loss which is accomplished by steam dump valves, relief valves and safety valve accordingly. Steam dump system is capable of dumping 70% of total steam in condenser of nuclear power plant through four dump valves. In addition, this system is also used with condition, when the plant is turned to operate with house-load. If the dump valves fail to open when they are expected to open for pressure control, the system pressure starts increasing. There are two relief valves provided for steam release into atmosphere. If relief valves fail to open due to loss power supply or control then system pressure starts increasing. There are two safety valves on each steam line header to provide over pressure protection. This system is used to release the surplus steam in the steam generator under the condition that the main steam isolation valve has not opened during the cold and hot start-up of reactor so as to maintain the steam pressure of steam generator within the required range.

The conventional control system consists of PID controller, permissive, interlocks and special functions implemented on PLC based control system which is beyond the access for plant operator. The switching and line-up of valves is incorporated depending on steam pressure conditions. This conventional control system is dependent on real time plant interfaced with PLC. In this research work, an offline modelling and design approach has been adopted for novel steam dump controller synthesis based on dynamic plant data and scheduling parameters, estimated using sophisticated functions formulated from design data of plant.

Some relevant literature is reviewed for modelling and control synthesis of steam dump control system. A mixed sensitivity based robust controller has been investigated for flight launcher in [1] in discrete time domain. Similarly, a robust H_∞ controller has been investigated for SISO systems in [2] using complex molecular functions. A research has been conducted for optimal and robust reactor power controllers design for nuclear research reactor and PWR-type nuclear power plants in [3]. The research has been extended for reactor power controller design of PHWR-type nuclear power plant in [4] using H_∞ optimization. A linear parameter varying (LPV) model based H_∞ controller has been proposed for pneumatic actuator in [5]. Further research has been executed for fractional order LPV modelling of canal control purposes in [6]. A robust fractional order PID controller has been evaluated for LPV dynamic mechatronic system in [7]. Therefore, concepts of fractional order modelling and robust fractional order PID controller has been arrived at, up to this literature survey. Now, the research has been further extended for fractional order vibratory system and a fractional order optimal LQR controller and state observer have been proposed in [8]. A research work has been explored for fractional order interval systems and controller design with uncertainties modelling in [9]. A H_∞ controller has been identified for fractional order systems in [10]. LMI based robust FOS controller has been synthesized for spatial control of large PHWR in [11]. Research on LMI of robust FOS controller has been extended for fractional order systems in [12]. These LMIs has proven a strong base for robust FOS controller extendable to other H_∞ controller as well. Comprehensive details of steam dump system for AP600 PWR nuclear power plant have been presented in [13-15]. The behaviour of various uncertain parameters has been identified in these workshop materials. Some of the parameters related to steam dump system is selected as scheduling parameters. The behaviour of steam pressure under some patent transients of steam dump system has been selected as benchmark for this research work [16]. A gain scheduled subspace model predictive control of reactor power has been addressed in [17].

Our proposed methodology is one step ahead in this direction to design a robust fractional order LPV switching H_∞ controllers for overall steam dump system of PWR-type nuclear power plant after considering a different PWR [13-16] and incorporating dynamic multi-scheduling parameters of cold leg coolant temperature coefficient of reactivity, hot leg coolant temperature coefficient, steam flow rate and turbine power for switching of controller for dump valves, relief valves and safety valves.

2. Modelling of Steam Dump System

The steam dump control system in PWR-type nuclear power plant is operated in two modes. One mode in which reactor power is less than 15% is steam pressure mode while the second mode in which reactor is greater than 15% is called reactor coolant average temperature mode as shown in Fig. 1 and Fig. 2 respectively.

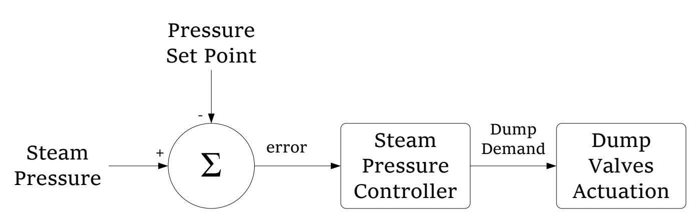

Fig. 1. Steam pressure control mode of steam dump system

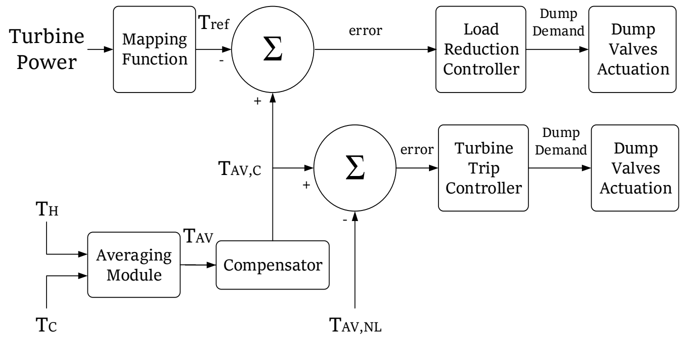

Fig. 2. Reactor coolant temperature mode of steam dump system

3. Linear Parameter Varying Modelling

In this research work, the model of steam dump is developed based on dynamic data of steam dump system called system identification method and some other parameters that are variable in nature with time affecting the dynamics of steam dump system called scheduling parameters ϕ_i. Such a system is called linear time varying system (LPV). The LPV system is basically a grid of LTI systems, if scheduling parameter is ϕ_i as shown in Fig. 3.

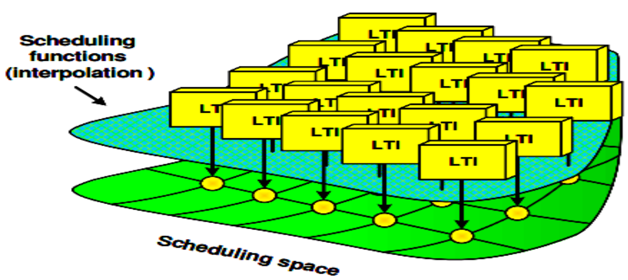

Fig. 3. General structure of linear parameter varying system

In steam dump system, the scheduling parameters are the function of time $\phi_i(t)$. So, these scheduling parameters are dynamic in nature that makes the control problem more challenging. The overall design scheme of Steam Dump Control System of PWR-type NPP is shown in Fig. 4.

3.1 Fractional Order Linear Parameter Varying Model

The fractional order MISO linear parameter model of steam dump system (FO-MISO-LPV-SDS) is described using a time domain differential operator D^{α} and D^{β} where α and β are the fractional order of differential operators associated with denominator and numerator polynomials respectively. If the dynamics of steam dump system is strongly dependent on transient cold leg coolant temperature coefficient of reactivity $T_C^{\rho_C}(t)$, hot leg coolant temperature coefficient $T_H^{\rho_H}(t)$, steam flow rate $Q(t)$ and turbine power $T_L(t)$ then the dynamic scheduling parameters are $\phi_i(t)$, $i = 1,2,3,4$. The FO-MISO-LPV-SDS model is estimated in polynomial fashion and the coefficients are the functions of $\phi_i(t)$.

Now, before proceeding towards the FO-MISO-LPV-SDS problem formulation, the dynamic scheduling parameters $\phi_i(t)$ are modeled as a function reactor power $P_R(t)$ and rate of change of reactor power $\dot{P}_R(t)$ based on nuclear reactor and SDS design data of AP600 PWR-type nuclear power plant in functional forms as Eqs. 1-4.

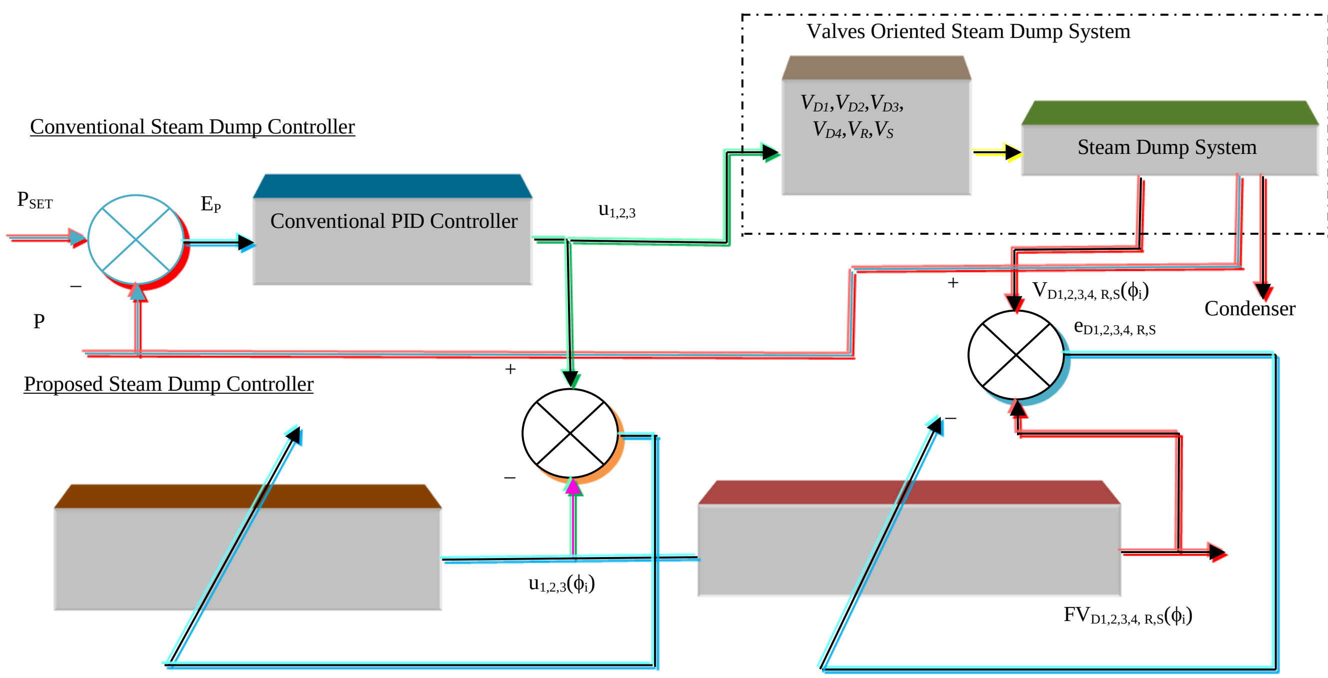

Fig. 4. Closed loop framework for fractional order steam dump control system design

$$\phi_1(t) = T_C^{\rho_C}(t) = f_1[P_R(t), \dot{P}_R(t)] \quad (1)$$

$$\phi_2(t) = T_h^{\rho_h}(t) = f_2[P_R(t), \dot{P}_R(t)] \quad (2)$$

$$\phi_3(t) = Q(t) = f_3[P_R(t), \dot{P}_R(t)] \quad (3)$$

$$\phi_4(t) = T_L(t) = f_4[P_R(t), \dot{P}_R(t)] \quad (4)$$

3.2 FO-MISO-LPV-SDS Model

If V_{Dj} (j=1,2,3,4) are the valve opening input signals of four dump valves as input signals and P is the steam pressure as an output signal then MISO FO-LPV-SDS model with dump valves and dynamic scheduling parameters FO-MISO-LPV-DV can be formulated Eq. 5.

$$P^D(t) = \frac{1}{A^D[q, \phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t)]}[\sum_{j=1}^{4} B_j^D(q, \phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t))V_{Dj}(t)] \quad \ldots(5)$$

where $A^D(.)$ and $B_j^D(.)$ are the denominator and numerator polynomials representing dynamics of the dump valves oriented SDS model with q as the inverse unit delay operator.

If V_{Rk} (k = 1,2) are the valve opening input signals of two relief valves as input signals and P is the steam pressure as an output signal then MISO FO-LPV-SDS model with relief valves and dynamic scheduling parameters FO-MISO-LPV-RV can be formulated Eq. 6.

$$P^R(t) = \frac{1}{A^R[q, \phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t)]}[\sum_{k=1}^{2} B_k^R(q, \phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t))V_{Rk}(t)] \quad \ldots(6)$$

where $A^R(.)$ and $B_k^R(.)$ are the denominator and numerator polynomials representing dynamics of the relief valves oriented SDS model.

If V_{Sl} (l=1,2) are the valve opening input signals of two safety valves as input signals and P is the steam pressure as an output signal then MISO FO-LPV-SDS model with safety valves and dynamic scheduling parameters FO-MISO-LPV-SV can be formulated Eq. 7.

$$P^S(t) = \frac{1}{A^S[q, \phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t)]}[\sum_{l=1}^{2} B_l^D(q, \phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t))V_{Sl}(t)] \quad \ldots(7)$$

where $A^S(.)$ and $B_k^S(.)$ are the denominator and numerator polynomials representing dynamics of the relief valves oriented SDS model.

If $\xi(t)$ is the noise signal associated with measured steam pressure signal $P(t)$, then the noise coupled steam pressure signal is designated as $P_\xi(t)$.

Now, the FO-MISO-LPV-DV model described in Eq. 5, can be formulated in fractional order differential equation form as Eqs. 8 and 9.

$$A^D(\phi_i(t), D^{\alpha_{AD}})P^D(t) = \sum_{i=1}^{4} B_i^D(\phi_i(t), D^{\beta_{Bi}})V_{Di}(t) \quad (8)$$

$$P_\xi^D(t) = P^D(t) + e_\xi^D(t) \quad (9)$$

where $e_\xi^D(t)$ is the noise signal associated with steam pressure signal when dump valves are operating.

Now, the FO-MISO-LPV-RV model described in Eq. 6, can be formulated in fractional order differential equation form as Eqs. 10 and 11.

$$A^R(\phi_j(t), D^{\alpha_{AR}})P^R(t) = \sum_{j=1}^{2} B_j^R(\phi_j(t), D^{\beta_{Bj}})V_{Ri}(t) \quad (10)$$

$$P_\xi^R(t) = P^R(t) + e_\xi^R(t) \quad (11)$$

where $e_\xi^R(t)$ is the noise signal associated with steam pressure signal when relief valves are operating.

Now, the FO-MISO-LPV-SV model described in Eq. 7, can be formulated in fractional order differential equation form as Eqs. 12 and 13.

$$A^S(\phi_l(t), D^{\alpha_{AS}})P^S(t) = \sum_{l=1}^{2} B_l^S(\phi_l(t), D^{\beta_{Bl}})V_{Sl}(t) \quad (12)$$

$$P_\xi^S(t) = P^S(t) + e_\xi^S(t) \quad (13)$$

where $e_\xi^s(t)$ is the noise signal associated with steam pressure signal when safety valves are operating.

If n_D and m_D are the number of coefficients of polynomials $A^D(.)$ and $B_j^D(.)$ then the polynomials can be defined in terms of coefficients for FO-MISO-LPV-DV as Eqs. 14 and 15.

$$A^D(\phi_i(t), D^{\alpha_{AD}}) = 1 + \sum_{n_D=1}^{N_D} a_{n_D}[\phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t)]D^{\alpha_{n_D}} \quad (14)$$

$$B^D(\phi_i(t), D^{\beta_{BD}}) = 1 + \sum_{m_D=1}^{N_D} b_{m_D}[\phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t)]D^{\alpha_{m_D}} \quad (15)$$

If n_R and m_R are the number of coefficients of polynomials $A^R(.)$ and $B_k^R(.)$ then the polynomials can be defined in terms of coefficients for FO-MISO-LPV-RV as Eqs. 16 and 17.

$$A^{R}(\phi_i(t), D^{\alpha_{AR}}) = 1 + \sum_{n_R=1}^{N_R} a_{n_R}[\phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t)] D^{\alpha_{n_R}} \quad (16)$$

$$B^{R}(\phi_i(t), D^{\beta_{RR}}) = 1 + \sum_{m_R=1}^{N_R} b_{m_R}[\phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t)] D^{\alpha_{m_R}} \quad (17)$$

If n_S and m_S are the number of coefficients of polynomials $A^S(.)$ and $B_l^S(.)$ then the polynomials can be defined in terms of coefficients for FO-MISO-LPV-SV as Eqs. 18 and 19.

$$A^{S}(\phi_i(t), D^{\alpha_{AS}}) = 1 + \sum_{n_S=1}^{N_S} a_{n_S}[\phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t)] D^{\alpha_{n_S}} \quad (18)$$

$$B^{S}(\phi_i(t), D^{\beta_{BS}}) = 1 + \sum_{m_S=1}^{N_S} b_{m_S}[\phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t)] D^{\alpha_{m_S}} \quad (19)$$

Since all the coefficients of FO-MISO-LPV-DV, FO-MISO-LPV-RV and FO-MISO-LPV-SV are dynamic in nature; therefore, there is no analytical solution. The problem is solved using numerical optimization and all the coefficients are dynamic in nature.

Each identified model FO-MISO-LPV-DV, FO-MISO-LPV-RV and FO-MISO-LPV-SV in FO-MISO-LPV-SDS model is evaluated using the best fitness test (BFT). Now, instead of formulating and repeating it separately for FO-MISO-LPV-DV, FO-MISO-LPV-RV and FO-MISO-LPV-SV, generic formulation is established. Therefore, BFT is computed by taking the difference of estimated output $\hat{y} = y_{\xi}(t) = y_{LPV}(t)$ from the system actual output y = $y_{actual}(t)$ as Eq. 20.

$$BFT = \max(1 - \frac{\|y - \hat{y}\|_2}{\|y - \bar{y}\|_2}, 0) \times 100\,\% \quad (20)$$

where $\bar{y} = \bar{y}_{\xi} = \bar{y}_{LPV}$ is the mean value of the output variable.

In following sections, FO-MISO-LPV-SDS model is restructured for RFO-LPV-SWH$_\infty$ controllers' synthesis is presented.

4. State space modeling of FO-LPV-SDS

Now for RFO-LPV-SWH$_\infty$ controllers' synthesis, FO-MISO-LPV-SDS model is required to be in state space form. Therefore, FO-MISO-LPV-SDS model is converted into fractional order LPV state space descriptor form as Eqs. 21 and 22.

$$ED^{\alpha}x(t) = A(\phi_1(t), \phi_2(t), \phi_3(t), \phi_4(t))x(t) + Bu(t) \quad (21)$$

$$y(t) = Cx(t) + Du(t) \quad (22)$$

4.1 Polytopic FO-MISO-LPV-SDS Model

Now, the fractional order LPV state space descriptor model is restructured in system matrix (*S*) as Eqs. 23-25.

$$ED^{\alpha}x(t) = A(t)x(t) + Bu(t) \quad (23)$$

$$y(t) = Cx(t) + Du(t) \quad (24)$$

$$\phi(t) = \eta_1 S_1 + \eta_2 S_2 \quad (25)$$

where S_1 and S_2 are system matrices at two vertices of polytope and η_1 and η_2 are the coordinates of polytope at two extreme values of uncertain scheduling parameter such that $\eta_1 + \eta_2 = 1$. One is defined at 100% reactor power while second is defined at 50% reactor power. This is accomplished to simplify the model and thereby for the controller design for case study and dynamic performance analysis purposes. Otherwise, there are number of models and number of controllers depending of convex decomposition.

If $p = \sqrt{-1}$ is the imaginary number, then S_1 and S_2 are system matrices at two vertices of polytope can be defined as Eqs. 26 and 27.

$$S_1 = \begin{bmatrix} A_1 + pE_1 & \vdots & B_1 \\ \cdots & \vdots & \cdots \\ C_1 & \vdots & \cdots \end{bmatrix} \quad (26)$$

$$S_2 = \begin{bmatrix} A_2 + pE_2 & \vdots & B_2 \\ \cdots & \vdots & \cdots \\ C_2 & \vdots & \cdots \end{bmatrix} \quad (27)$$

4.2 Affine Parameter Dependent FO-MISO-LPV-SDS Model

Now, the Polytopic FO-MISO-LPV-SDS state space model is restructured, if the state space matrices have affine dependence on ϕ_i as Eqs. 28-32.

$$A(\phi_i) = A_0 + \phi_i A_1 \quad (28)$$

$$B(\phi_i) = B_0 + \phi_i B_1 \quad (29)$$

$$C(\phi_i) = C_0 + \phi_i C_1 \quad (30)$$

$$D(\phi_i) = D_0 + \phi_i D_1 \quad (31)$$

$$E(\phi_i) = E_0 + \phi_i E_1 \quad (32)$$

where A_0, B_0, C_0, D_0, E_0 and E_0 are parameter independent matrices, therefore, the affine parameter dependent model can be described as Eqs. 33 and 34.

$$ED^{\alpha}x(t) = A(\phi_i)x(t) + Bu(t) \quad (33)$$

$$y(t) = Cx(t) + Du(t) \quad (34)$$

Now, the overall SYSTEM matrix is given as Eq. 35.

$$S(\phi_i) = S_0 + \phi_i S_1 \quad (35)$$

4.3 Trajectory Formulation of Scheduling Parameters

Now, the trajectory of scheduling parameters in polytope can be formulated as Eqs. 36 and 37.

$$\phi_i(t) = \sum_{r=1}^{N} \chi_r(t)\phi_i \quad (36)$$

$$\sum_{r=1}^{N} \chi_r(t) = 1 \quad (37)$$

where $\chi_r(t)$ are computed by convex decomposition. *N* represents number of vertices of polytope.

5. Design of robust fractional order LPV H∞ (RFO-LPV-H∞) controller

5.1 Conventional Steam Dump Control System

The dump valves oriented conventional steam dump control system is shown in Fig. 5 while the overall steam dump control system with dump, relief and safety valves is shown in Fig. 6.

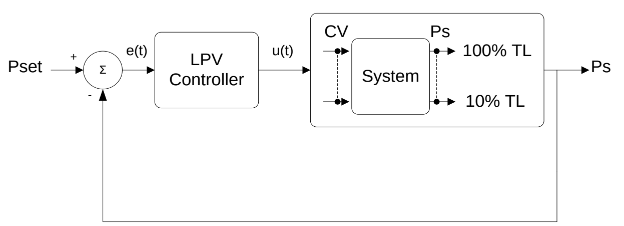

Fig. 5. Closed loop dump valves oriented steam dump control system

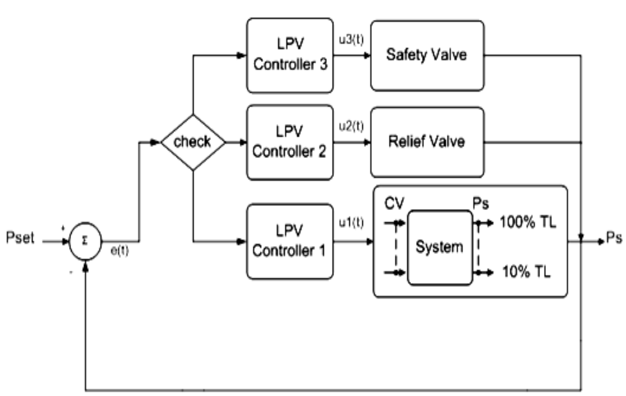

Fig. 6. Overall design of closed loop steam dump control system

5.2 Control System Design Constraints

The steam dump control system has the following constraints on scheduling parameters, inputs, output and switching scheme of controllers:

$$\begin{cases} \dot{\phi}_4(t) = \dot{T}_L(t) < \dot{Q}(t) & for \quad t > 0 \\ \dot{\phi}_3(t) = \dot{Q}(t) < \dot{T}_C^{\rho_C}(t) & for \quad t > 0 \\ \dot{\phi}_1(t) = \dot{T}_C^{\rho_C}(t) < \dot{T}_H^{\rho_H}(t) & for \quad t > 0 \\ \dot{\phi}_2(t) = \dot{T}_H^{\rho_H}(t) \le \dot{T}_{H(Upper)}^{\rho_H}(t) & for \quad t > 0, \\ \hat{y}_D(t) = P_{\xi}^{D}(t) < \hat{y}_R(t) & for \quad t > 0, \\ \hat{y}_R(t) = P_{\xi}^{R}(t) < \hat{y}_S(t) & for \quad t > 0, \\ \hat{y}_S(t) = P_{\xi}^{S}(t) \le P_{\max} & for \quad t > 0, \\ u(t) = u_1(t) = V_{Dj}(t) & for \quad t = t_1 \ge t_{Set-Point}^{D}, \\ u(t) = u_2(t) = V_{Rk}(t) & for \quad t = t_2 \ge t_{Set-Point}^{R} > t_1, \\ u(t) = u_3(t) = V_{Sl}(t) & for \quad t = t_3 \ge t_{Set-Point}^{S} > t_2 \end{cases}$$

5.3 Two Port Formulation and Weights Selection

The proposed framework of closed loop output feedback RFO-LPV-H∞ controller is shown in Fig. 7. W_p, W_M and W_U are the adjustable fractional order weights associated with controller input, SDS output and control valves oriented steam dump system controller output respectively.

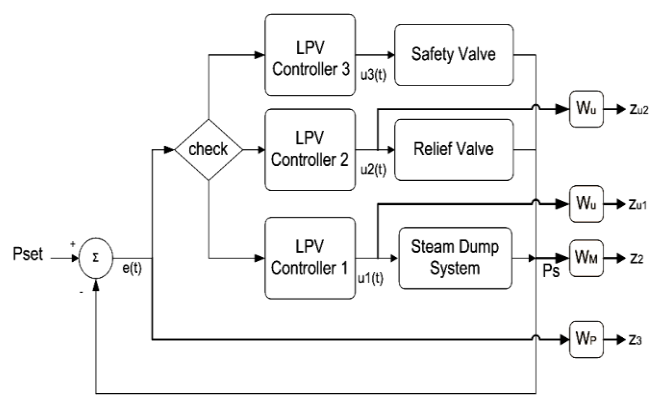

Fig. 7. Framework of closed loop FO-LPV-RSW-H∞ steam dump system

Now, the two port perturbed model of Fig. 7 can be formulated as Eqs. 38-40 [4].

$$D^{\alpha}x(t) = A(\phi_i(t))x(t) + B_u u(t) + B_w w(t) \quad (38)$$

$$y(t) = C_y x(t) + D_{yu}u(t) + D_{yw} w(t) \quad (39)$$

$$z(t) = C_z x(t) + D_{zu}u(t) + D_{zw} w(t) \quad (40)$$

Now, the state space of $T_{zw}(s)$ is given as Eqs. 41-42.

$$D^{\alpha}x(t) = A(\phi_i(t))x(t) + B_w w(t) \quad (41)$$

$$z(t) = C_z x(t) + D_{zw} w(t) \quad (42)$$

The adjustable fractional order weights W_p, W_M and W_U are selected based on the closed loop performance of proposed scheme and therefore can be formulated as Eqs. 43-45.

$$W_p(s) = \frac{a_\mu^P}{s^{v_P} + b_\mu^P} \quad (43)$$

$$W_u(s) = \frac{a_\mu^u s^{v_{u_1}} + 1}{b_\mu^{u_1} s^{v_{u_2}} + b_\mu^{u_2}} \quad (44)$$

$$W_M(s) = \frac{a_\mu^M}{b_\mu^M s^{v_M} + 1} \quad (45)$$

where v is the order associated with respective weight.

5.4 Design Framework of Fractional Order H∞ Controller

The problem is to find control law $u(t)$ with output feedback configuration as Eq. 46.

$$u(t) = K(\phi_i)y(t) \quad (46)$$

H-Infinity norm of robust fractional order H_∞ controller is given as Eq. 47.

$$\|T_{zw}(s)\|_{H_\infty} < \gamma \quad (47)$$

The H_∞ cost function can be defined as:

$$J_{H_\infty} = \int_0^\infty z^T(\tau)z(\tau) - \gamma^2 w^T(\tau)w(\tau)d\tau < 0 \quad (48)$$

5.5 Design of LMIs for Fractional Order H∞ Controller Optimization

Now the problem is to formulate Linear Matrix Inequalities (LMIs) satisfying Eqs. 41, 42 and 47 for FO-MISO-LPV-SDS model so that 0 < α < 1 and 1 < α < 2 [8].

$$\begin{bmatrix} \bar{r}A^T(\phi_i)X + Xr A(\phi_i) & XB_w & \bar{r}C_z^T \\ B_w^T X & -\gamma^2 I & D_{zw}^T \\ rC_z & D_{zw} & -I \end{bmatrix} < 0 \quad (49)$$

If and if there exist X = X* satisfying Eqs. 47 and 49.

For the fractional order model described in Eqs. 38-40 is stable, if and only, the following inequality [9].

$$\bar{r}A^T(\phi_i)X + Xr A(\phi_i) < 0$$

This satisfies the matrix given in the following Eq. 50.

$$rA(\phi_i) = e^{(1-\alpha)p\frac{\pi}{2}} A(\phi_i) \quad (50)$$

proves that for 0 < α < 1:

$$\|T_{zw}(s)\|_{H_\infty} = \sqrt{\gamma_{optimal}} \quad (51)$$

And, for 1 < α < 2 upper bound on $\|T_{zw}(s)\|_{H_\infty}$:

$$\|T_{zw}(s)\|_{H_\infty} \le \sqrt{\gamma_{optimal}} \quad (52)$$

6. Design of Switching Scheme for Fractional Order *H∞* Controller

Since there are three sets of valves for steam dump, steam relief and safety of SDS. Therefore, switching of controller is required for their actuation logic. The switching and actuation of dump, relief and safety valves controllers are accomplished as per steam dump control system design constraints.

Healthy dump values;

$$u_1(t) = K_1(\phi_i)y(t) = K_D(\phi_i)y_D(t) \quad for\ t = t^D_{Set-Point}$$

Faulty dump values;

$$u_2(t) = K_2(\phi_i)y(t) = K_R(\phi_i)y_R(t) \quad for\ \ t = t^R_{Set-Point} = t^D_{Set-Point} + t_{d_1}$$

Faulty relief values;

$$u_3(t) = K_3(\phi_i)y(t) = K_S(\phi_i)y_S(t) \quad for\ \ t = t^S_{Set-Point} = t^R_{Set-Point} + t_{d_2}$$

where t_{d_1} and t_{d_2} are relief and safety valves actuation delayed which are measured from the real time process of SDS.

7. Evaluation of design parameters

Now, in this section, numerical values of design parameters are presented. Equation (1) through Equation (29) is solved numerically using a specialized dedicated program developed in MALTAB environment. All calculations are performed in detail. Now, in this research work, sample results of FO-MISO-LPV-DV model at first vertex of polytope at $P_R(t)$ =100% reactor power and $\dot{P}_R(t)$ =10% / min maximum rate of change of reactor power are presented below.

$$A^D_{First-Polytope} = \begin{bmatrix} -0.0004311 & 0.000125 & 0.008634 \\ -0.0032752 & -0.9747 & -7.56829 \\ -0.0256758 & 5.356858 & -0.88542 \end{bmatrix}$$

$$B^D_{First-Polytope} = \begin{bmatrix} 2.476e-08 & 1.34e-05 & 1.3854e-06 & 1.2456e-08 \\ -0.00056422 & -0.03465 & -0.0056875 & -3.763e-05 \\ -0.00067452 & 0.0003864 & -0.0049765 & -0.0064785 \end{bmatrix}$$

$$C^D_{First-Polytope} = [1476 \quad -9.348 \quad 3.736]$$

$$D^D_{First-Polytope} = [0 \quad 0 \quad 0 \quad 0]$$

Similarly, FO-MISO-LPV-DV model at second vertex of polytope or any vertex of polytope can be computed. Rest of FO-MISO-LPV-RV and FO-MISO-LPV-SV models are computed through the same developed program in MATLAB.

As system shows uncertain behavior during plant operation, so the system is modeled by appending fractional order weighting filters for controller design purposes.

The fractional order weights W_p, W_M and W_U for FO-MISO-LPV-DV model are optimized as Eqs. 53, 54 and 55 respectively.

$$W_p(s) = \frac{0.004}{s^{0.5} + 0.03} \tag{53}$$

$$W_u(s) = \frac{0.00002s^{0.25} + 1}{0.3s^{0.25} + 18000} \tag{54}$$

$$W_M(s) = \frac{0.007}{18s^{0.5} + 1} \tag{55}$$

Similarly, rests of FO-MISO-LPV-RV and FO-MISO-LPV-SV weights are optimized.

The Robust Fractional Order Linear Parameter Varying Dump Valve H_∞ (RFO-LPV-DVH_∞) controller is comprised of $K_{D1}(s)$, $K_{D2}(s)$, $K_{D3}(s)$ and $K_{D4}(s)$ four fractional order transfer functions given as Eqs. 56-60.

$$U_1(s) = [K_{D1}(s) \quad K_{D2}(s) \quad K_{D3}(s) \quad K_{D4}(s)]Y(S) \tag{56}$$

$$K_{D1}(s) = \frac{1204.78(s^{0.75} + 20.3)}{(s^{0.5} + 15.3)(s^{0.25} + 800.56)} \tag{57}$$

$$K_{D2}(s) = \frac{1371.56(s^{0.75} + 85.45)}{(s^{0.5} + 89.2)(s^{0.25} + 765.75)} \tag{58}$$

$$K_{D3}(s) = \frac{900.12(s^{0.75} + 17.56)}{(s^{0.5} + 77.89)(s^{0.25} + 645.55)} \tag{59}$$

$$K_{D4}(s) = \frac{385.12(s^{0.75} + 11.55)}{(s^{0.5} + 19.45)(s^{0.25} + 589.38)} \tag{60}$$

The optimal value of H_∞ norm for Robust Fractional Order Linear Parameter Varying Dump Valve H_∞ (RFO-LPV-DVH_∞) controller for FO-MISO-LPV-DV model is found below:

$$\|T_{zw}(s)\|_{H_\infty} = 1.01264$$

For α =0.5, the H_∞ performance is calculated as $\gamma = 1.0254$.

The optimal cost of RFO-LPV-DVH_∞ controller is $J_{RFO-LPV-DVH_\infty} = 0.455$.

Similarly, rests of the values of H_∞ norm for FO-MISO-LPV-RV and FO-MISO-LPV-SV controllers are optimized.

8. Performance Analysis

In this section, performance of proposed fractional order multi-scheduling parameters based LPV open and closed loop SDS is evaluated. The steam dump control system is designed for 600 MWe PWR-type nuclear power plant and valid for AP600 only.

Once the model parameters are identified for SDS, the estimated model is validated and tested for random input. Now, the identified model is tested by applying a bounded random input within the model validity bounds with zero initial conditions and coupled with random noise of 10^{-3}. The test program is run to get system output *y*. Now, the system input *u* and measured output *y* are applied to test program and the dynamics of SDS is predicted against random signal in order to validate the model. The test program runs successfully and BFT factor is 99.95%. The simulation result for 50 samples is shown in Fig. 8.

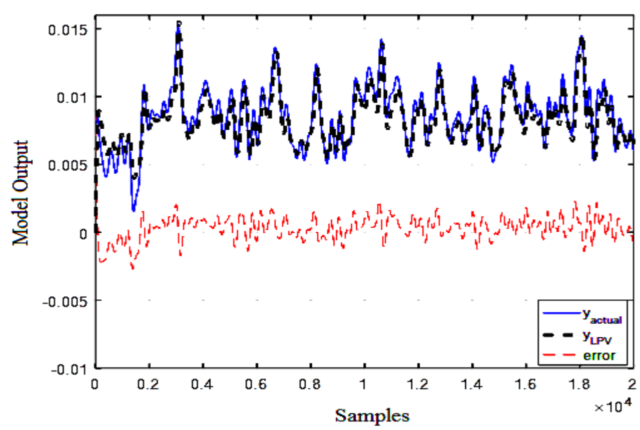

Fig. 8. Estimation and validation of LPV steam dump system MOD

The proposed closed loop fractional order multi-scheduling parameters SDS is simulated and tested for two ramp load reduction cases. One from 100% turbine load to 0% and second is from 50% turbine load to 30%.

In this research work, large load reduction transient from 100% to 0% is presented. The turbine load is reduced from 100% to 0% at design rate of rate 10%/min. As a result, the system pressure is increased from 55 kg/cm^2 to 82 kg/cm^2, which is shown in Fig. 9. This increase in pressure is highly undesirable. So, the reference input of the signal is set-point steam pressure in normalized form and the behavior in actual units is shown in Fig. 9. The RFO-LPV-DVH$_\infty$ controller copes this pressure surge and brings the system pressure back to safe limit. It gives the optimal performance as shown in Fig. 9 in zoomed form.

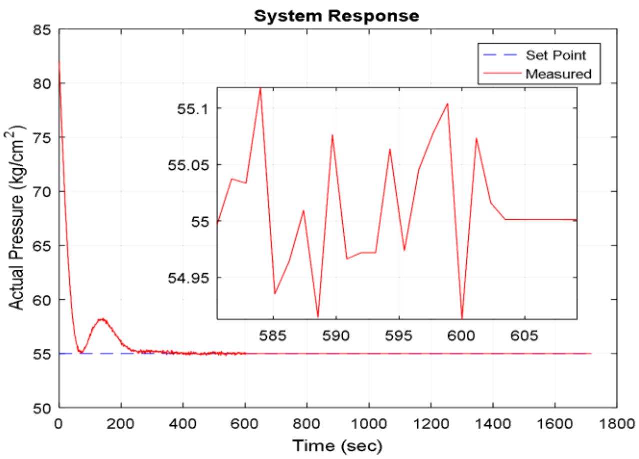

Fig. 9. Steam pressure response in tracking set-point

To control the system pressure, the steam dump valves V_1, V_2, V_3 and V_4 actuate as shown in Fig. 10 to Fig. 13 at vertex 1 of the polytope. The process delay in actuation of dump valves is measured and found 200 seconds. All the dump valves gradually open and jump to 100% opening demand and ultimately stabilize in a steady position.

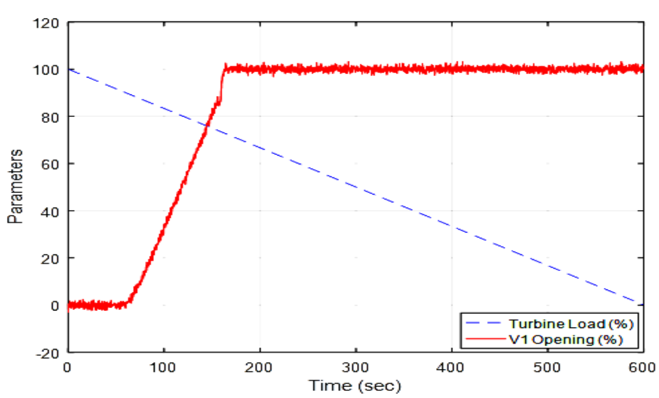

Fig. 10. Steam dump valve-1 actuation with turbine load at first vertex of polytope

Now, the performance of relief valves is tested, in a turbine load reduction test, when the dump valves fail to actuate. Therefore, the system steam pressure rises and increases from 82 kg/cm^2 to 84.7 kg/cm^2 as shown in Fig. 14. This rise in steam pressure is undesirable. To control the system pressure, both steam relief valves V_{R1} and V_{R2} actuate as shown in Fig. 15 at vertex 1 of the polytope. Separate signals are provided for each V_{R1} and V_{R2} to actuate in order to provide more reliability by increasing the redundancy level and hence the system safety is ensured. The Robust Fractional Order Linear Parameter Varying Relief Valve H_∞ (RFO-LPV-RVH$_\infty$) controller opens the atmosphere relief valves and hence controller copes this problem, minimizes the H_∞ norm and maintains the system pressure at 82 kg/cm^2 as Fig. 16.

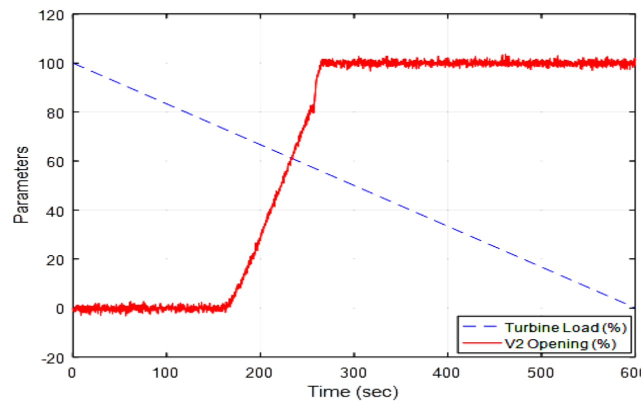

Fig. 11. Steam dump valve-2 actuation with turbine load at first vertex of polytope

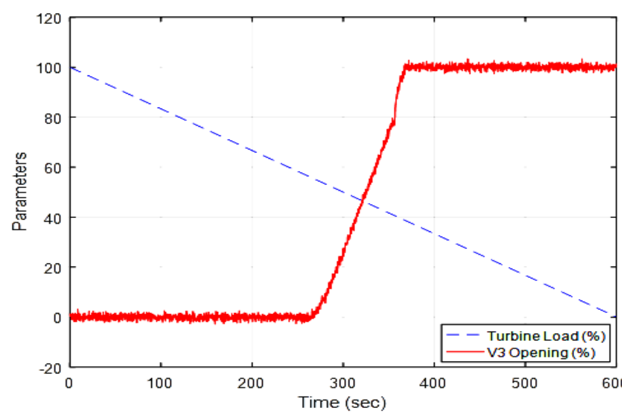

Fig. 12. Steam dump valve-3 actuation with turbine load at first vertex of polytope

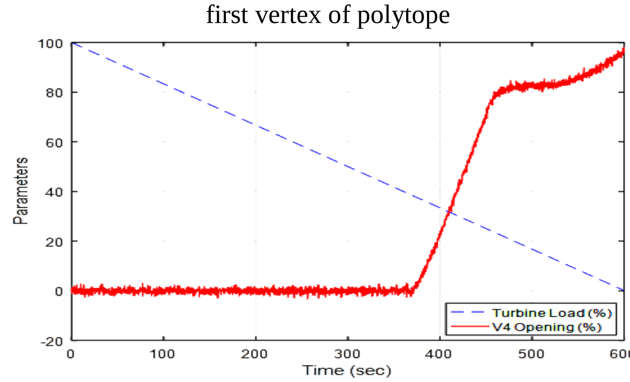

Fig. 13. Steam dump valve-4 actuation with turbine load at first vertex of polytope

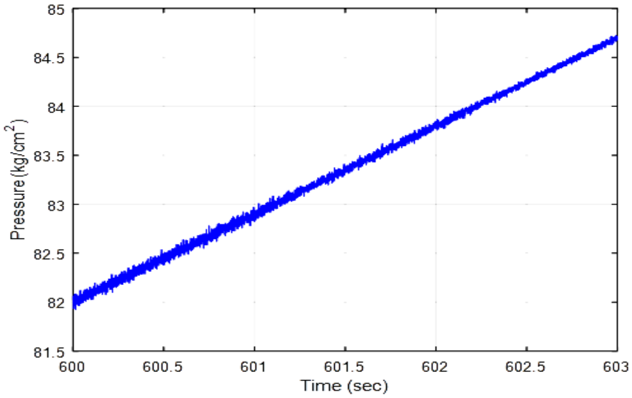

Fig. 14. Steam pressure variation due to fault

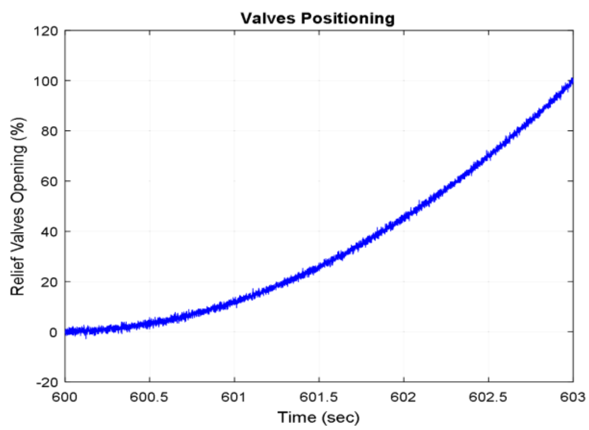

Fig. 15. Valve actuation at first vertex of polytope

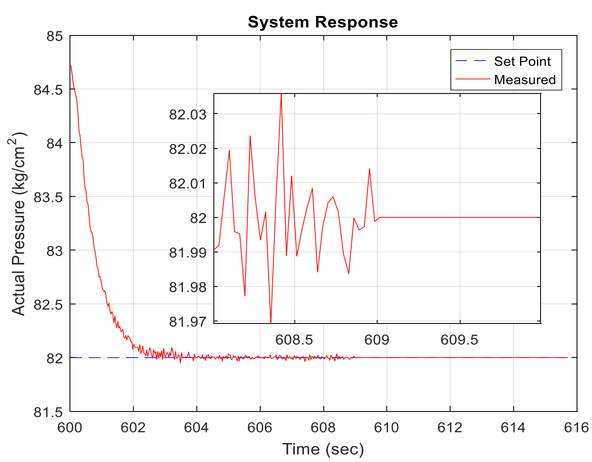

Fig. 16. Steam pressure compensation with relief valves

Similarly, the performance of safety valves is tested, in a turbine load reduction test, when the both dump and relief valves fail to actuate. The Robust Fractional Order Linear Parameter Varying Safety Valve H_∞ (RFO-LPV-SVH_∞) controller opens the atmosphere safety valves and hence controller copes this problem, minimizes the H_∞ norm and maintains the system pressure exactly in a similar manner like relief valves.

9. Conclusion

A highly nonlinear fractional order linear parameter varying model has been proposed in this research work for dynamic analysis of steam dump system and controller design for the secondary side of PWR- type nuclear power plant. The novel fractional order model of steam dump system has multi-scheduling LPV framework and the coefficients of fractional order model parameters have been estimated using fractional linear parameter varying least square technique. Highly nonlinear dynamic multi-scheduling parameters based problem has been first time attempted in this research work for steam dump system in a PWR type nuclear power plant. Steam dump system has switch mechanism for valves selection and actuation logics. This has adopted the more challenging situation in multi input single out parallel computing framework. The estimated BFT factor for the proposed model is 99.95%. A robust fractional order LPV switching H_∞ controller has been proposed for dump valves, relief valves and safety valves. The performance of proposed controller has been evaluated under expected steam bypass transients and fault condition of steam dump valves and relief valves. Nonlinear functions based multi-scheduling parameters and fractional order LPV MISO model of SDS has been evaluated at first and second vertex of polytope and has a dynamic feature to evaluate any vertex of polytope for any power level and rate of change of power level. The model has an applicability to append with addition of more scheduling parameters of primary and secondary side of nuclear power plant for further enhancement of robustness. The LMIs has proven a strong base for fractional order LPV switching H_∞ controller extendable to other H_∞ controller as well with any other optimization algorithm. The performance of closed loop steam dump system in fractional order linear parameter varying structure is found robust and within design bounds for all input and output variables at first and second vertex of polytope. Hence, a quite satisfactory and a successful realization have been accomplished.

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