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Modified Hybrid Grey Model (1,1) to Forecast Cellular Subscribers

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ABSTRACT

This study develops MHGM (1,1) (Modified Hybrid Grey Model) which is the combination of two models first one is improved GM (1,1), this model consists of optimization of initial and background values and other is concave EDDGM (1,1) (Dynamic Discrete Grey Model) termed, in this model equal division technique is applied to fit the concavity of cumulative sequence and after that created dynamic average value and on the basis of that dynamic average value dynamic discrete GM (1,1) model is established and by the gradual heuristics method or the dichotomy approach the initial equal division number is obtained. We have fixed equal division number 'n' between 0 and 1in MHGM (1,1). For forecasting of starting half years we use $y^{(0)}$(m) as initial condition of model in time restored function and also multiply by a factor $e^{-b}{}_1$ to adjust the model. This model has applied without solving by heuristics or dichotomy method. Subscribers of cellular networks increase day by day in Pakistan; cellular industry has total five networks in Pakistan. In this paper data of three cellular networks subscribers that are Mobilink, Ufone and Zong have taken as application of models and it has been proved by using mean absolute percentage error that the forecast accuracy of MHGM (1,1) is better than GM (1,1) (Grey Model) and improved grey model (1,1).

Key Words: **Improved Grey Model, Concave Dynamic Discrete Model, Cellular Subscribers, Equal Division Number.**

1. INTRODUCTION

Now-a-days it is easy to make and receive a call at any time and any place by the advent of mobiles phone. In this info age; many people want to approach data while on travelling, instead using a stationary terminal which cannot be available at every place. Portable computers can directly connect to a cellular network mostly for those people who travel a lot. Through those portable computers faxes, e-mail and other forms of data can be sent and received by the availability of cellular network. Pakistan mobile industryis growing in present situations. Pakistan is one of the biggest cellular network user, approximately more than 134 million use mobile phone. In 2008 Pakistan included the world's fastest growing telecommunications market. The ARPU

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(Average Revenue Per User) per month of cellular mobile segment of the country increased to Rs. 440/- in 2015 which is greater in the comparison of Rs. 432/- in 2014. The PTA's (Pakistan Telecommunication Authority) BVS (Biometric Verification System) campaign give the data of right subscriber through which PTA derive correct ARPU in Pakistan. PTA plays an important role in increasing the growth rate of industry. After initiating of 3G/4G in Pakistan the mobile industry has felt basic structural changes. The operators invested 1.8 billion dollars on relevant infrastructure and increased license fee in 2014. Pakistan cellular industry consist of five networks that are Mobilink, Ufone, Zong, Telenor and Warid. The subscriber of cellular network increases day by day in Pakistan so in these situations the questions produce that: How many cellular subscribers will increase in future? How can the decision maker make the future plan based on small set of data? This leads us to grey theory which was given by Chinese professor Deng Julong in 1982. The Grey theory has been used in many areas as in electric engineering, transportation, economics, education etc. through Grey theory any one can encounter easy calculation and better results. In grey theory forecast is done on K e" 4 points. GM (1,1) has a number of advantages but its boundaries are limited. GM (1,1) is used for short term forecasting and it is best fit for the development coefficient a d" 0.5 and for little bit variation. For better forecasting; many methods have developed.

The grey theory has two aspect of information one is known and other information is not known. There are many works has presented on the accuracy of said model. Erdal Kayacan, Baris Ulutas, Okyay Kaynak gave MGM (1,1) by using Fourier series in time series [1]. An optimized nonlinear GBM (1,1) (Grey Bernoulli Model) was given by which parameters optimization of that model is suggested a combination of optimization problem and is performed on LINGO software and the model is used to forecast the per year industrial waste water rate of 31 state of China's [2]. The idea of omni-direction forecasting of non-equip gap data was given by improving the inverse accumulating generator operator [3]. It was forecasted number of internet users and number of online game players and Taiwan's revenue from 2003-2010 through GM (1,1) [4]. Wein Zhou, Jian-Min He introduced GGM (1,1) (Generalize Grey Model) on the basis of GM (1,1) and DDGM (1,1) in that study of four estimate approaches of stepwise ratio in generalize GM are given [5]. It was studied AGM (1,1) (Adaptive Grey Model)for the solution of wafer level packaging process and showed that this process give effective results with small data and it can be improved wafer level packaging process [6], and it was also explained that through perturbation bound parameters of GM (1,1) will change large when sample size of sample is large, so if sequence greater than or equal to zero, checking of quasi smooth and exponential condition is satisfied by input sequence then the original GM can achieve good prediction although the samples will be large [7]. It was told in study that what kind of factors affecting the tourist flow in china and an improved GM (1,1) was given which has a high prediction accuracy then the traditional GM (1,1) [8]. Huang CY, LU CY and Chen Cl this work is on GM (1,1) and nonlinear GBM and it is suggested that GM (1,1) is good for slow rate of change of data whereas nonlinear GBM (1,1) is good for drastically rate of change of data [9]. Ersi Liu, Qiangqiang Wang, Xinran Ge and Wei Zhou has shown two models that is the concave and convex DDGM (1,1) in these models equal division numbers are used by using two kind of approaches first is gradually heuristics method and other one is dichotomy method and eventually through conflict events in the urbanization process in china it was proved that DDGM (1,1) has higher accuracy than Discrete, optimal and GM (1,1) [10]. In another work that was put to forecast the growth rate of renewable energy consumption in China, in this work three model that is GM (1,1), nonlinear grey Bernoulli (1,1) and grey verhulst model is compared with each other and it can be show that grey verhulst model has greater accuracy than the two model the accuracy and fitness of models are also compared by regression analysis [11].

2. METHODOLOGY

First GM (1,1) and improved GM (1,1) will be applied and then compare results with MHGM (1,1). Following are steps of these three models.

2.1 GM (1,1)

Let $y^{(0)} = \{y^{(0)}(1), y^{(0)}(2), \ldots, y^{(0)}(m)\}$,

$Y^{(1)} = \{y^{(1)}(1), y^{(1)}(2), \ldots, y^{(1)}(m)\}$,

$Y^{(1)}(l) = \sum_{i=1}^{l} Y^{(0)}(i)$ and

$\overline{x}^{(1)}(l) = 0.5\, y^{(1)}(l-1) + 0.5\, y^{(1)}(l)$ then

$Y^{(0)}(l) + b_1 \overline{X}^{(1)}(l) = b_0$

Where 'b_1' stands for developing coefficient and 'b_0' stands for grey input action.

Time responded formula is $\hat{Y}^{(1)}(l+1) = \left[Y^{(0)}(1) - \frac{b_0}{b_1}\right] e^{-b_1 l} + \frac{b_0}{b_1}$, $l = 1,2,3,\ldots,m$

Restored function is $\hat{Y}^{(0)}(l) = \left[y^{(0)}(1) - \frac{b_0}{b_1}\right](1 - e^{b_1}) e^{-b_1 (l-1)}$, $l = 1,2,3,\ldots,m$

2.2 Improved GM (1,1)

Let $Y^{(0)} = \{y^{(0)}(1), y^{(0)}(2), \ldots, y^{(0)}(m)\}$,

$Y^{(1)} = \{y^{(1)}(1), y^{(1)}(2), \ldots, y^{(1)}(m)\}$,

$Y^{(1)}(l) = \sum_{i=1}^{l} y^{(0)}(i)$

$\overline{x}^{(1)}(l) = y^{(1)}(l) + \frac{y^{(0)}(l)}{\ln y^{(0)}(l) - \ln y^{(0)}(l-1)} - \frac{[y^{(0)}(l)]^2}{[y^{(0)}(l) - y^{(0)}(l-1)]}$ then

$Y^{(0)}(l) + b_1 \overline{X}^{(1)}(l) = b_0$

Where 'b_1' stands for developing coefficient and 'b_0' stands for grey input action.

Time response function is, $\hat{Y}^{(1)}(l+1) = [y^{(1)}(m) - \frac{b_0}{b_1}] e^{-b_1 (l-n)} + \frac{b_0}{b_1}$, $l = 0,1,2,\ldots,m-1$

Restored function is $\hat{Y}^{(0)}(l+1) = \hat{y}^{(1)}(l+1) - \hat{y}^{(1)}(l)$, $l = 0,1,2,\ldots, n-1$

2.3 Modified Hybrid Grey Model (1,1)

This study develops MHGM (1,1) which is combination of two models, background value is taken from improved GM (1,1) and the value of restored function is taken from EDDGM (1,1) and the value of 'n' which is equal division number is fixed between 0 and 1 with the help of 'r' value which represents number of years the model will be forecast without solving by heuristics or dichotomy method. For forecasting of starting half years we use $y^{(0)}(m)$ as initial condition of model in time restored function and also multiply by e^{-b_1} to adjust the model because this model is combination of two methods and in improved GM (1,1) we use $y^{(1)}(m)$ in time responded function for forecast of remaining final years same time restored function of EDDGM (1,1) is used.

(1) Let the data from original series

$y^{(0)} = \{y^{(0)}(1), y^{(0)}(2), \ldots, y^{(0)}(m)\}$

(2) Apply AGO (Accumulated Generating Operation) to obtain $Y^{(1)}$

$Y^{(1)} = \{y^{(1)}(1), y^{(1)}(2), \ldots, y^{(1)}(m)\}$

Where , $Y^{(1)}(l) = \sum_{i=1}^{l} Y^{(0)}(i)$, $l = 1,2,3\ldots,m$

(3) Construction of grey differential equation:

$$\frac{dY^{(1)}}{dt} + b_1 Y^{(1)} = b_0$$

Where 'b_1' stands for developing coefficient and 'b_0' stands for grey input action.

The above differential equation can be written:

$$Y^{(0)}(l) + b_1 \overline{X}^{(1)}(l) = b_0 \, \forall l = 2,3,\ldots,m$$

$\overline{X}^{(1)}(l)$ is background value and is represented:

$$\bar{x}^{(1)}(l) = y^{(1)}(l) + \frac{y^{(0)}(l)}{\ln y^{(0)}(l) - \ln y^{(0)}(l-1)} - \frac{[y^{(0)}(l)]^2}{[y^{(0)}(l) - y^{(0)}(l-1)]}$$

Parameters 'b_1' and 'b_0' will obtain from least square method.

(4) Time responded function can obtain by computational method of the difference equation Then time restored function for MHGM (1,1) model is:

$$\hat{y}^{(0)}(l+1) = \left(\frac{-2nb_1}{2n - b_1(n+1)}\right)\left(y^{(0)}(m) - \frac{b_0}{b_1}\right)\left(\frac{2n - b_1(n+1)}{2n + b_1(n-1)}\right)^l e^{-b_1}$$

$l = 0,1,2,\ldots,r$, r ≥ ½ then

$$\hat{y}^{(0)}(l+1) = \left(\frac{-2nb_1}{2n - b_1(n+1)}\right)\left(y^{(0)}(1) - \frac{b_0}{b_1}\right)\left(\frac{2n - b_1(n+1)}{2n + b_1(n-1)}\right)^l$$

$$n = \frac{1}{r}, \frac{2}{r}, \frac{3}{r}, \ldots, \frac{r}{r+1}$$

where n is number of step size taken, r is number of years which will be forecasted.

2.4 Measurement of Forecast Accuracy

Then to check forecast accuracy of these three models MAPE (Measurement of Forecast Accuracy) is computed, lower value of error shows higher accuracy of model.

Formula of MAPE is as follows

$$MAPE = \frac{1}{m} \sum_{i=1}^{m} \left| \frac{y_i - \hat{y}_i}{y_i} \right| \times 100\%$$

3. NUMERICAL EXAMPLE

In this study data of total subscribers of three networks that are Mobilink, Ufone and Zong in Pakistan are taken to forecast. Table1 show from 2009-2017 original data of three networks subscribers taken from and Fig. 1 shows the graph of original data.

TABLE 1. DATA OF THREE NETWORK SUBSCRIBERS FROM 2009-2017 IN PAKISTAN

Years	2009-2010	2010-2011	2011-2012	2012-2013	2013-2014	2014-2015	2015-2016	2016-2017
Subscribers	58455936	64839641	76687678	82847013	90318111	73336551	84203520	99185598

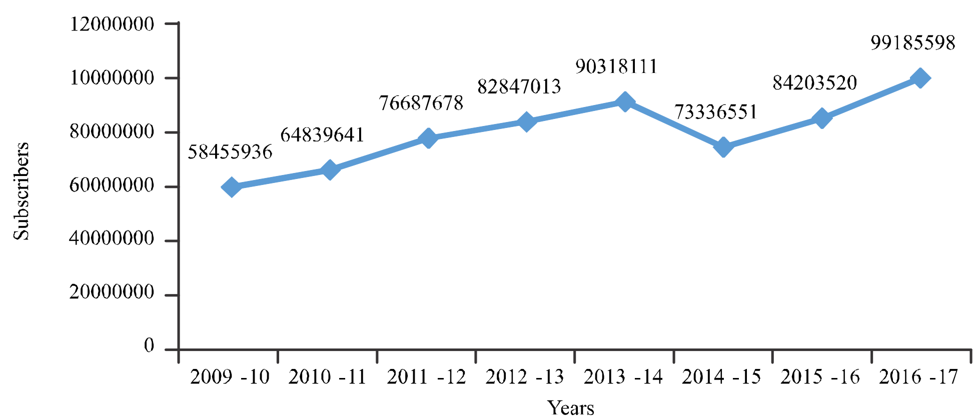

FIG. 1. GRAPH OF ORIGINAL DATA OF THREE NETWORKS SUBSCRIBERS FROM 2009-2017

Developing coefficient and grey action parameter of GM (1,1) are b_1= -0.118730618 and b_0 =55243558.46 and developing coefficient of improved MG (1,1) and MHGM (1,1) are b_1=-0.11858714 and b_0=55352770.52.

The results of these three models are shown in Table 2. Then MAPE is computed to compare three models in Table 3. Microsoft excel is used to solve these calculations. Further Fig. 2 shows graphically comparison between three models and actual data of model.

3.1 Discussion

Forecasting results of these three models in Table 2 it can be shown that MHGM (1,1) gives better results in comparison of other two models. MAPE in Table 3 of MHGM (1,1) is 0.1367% less than the rest of two models which are 0.1893and 0.1905% shows good forecasting results of MHGM (1,1) and this thing can be seen in Fig. 2 that MHGM (1,1) gives close approximation to original value than other two models.

TABLE 2. RESULTS OF THREE MODELS

Years	Original Data	GM (1,1)	Improved GM (1,1)	MHGM (1,1)
2009-2010	58455936	58455936	58455936	58455936
2010-2011	64839641	66008129.83	66094823.84	53963944.21
2011-2012	76687678	74327270.28	74452259.89	64433578.36
2012-2013	82847013	83694889.12	83827248.28	74807086.07
2013-2014	90318111	94243128.24	94382730.15	85832368.85
2014-2015	73336551	106120783.6	106267352.6	93542680.85
2015-2016	84203520	119495404.4	119648480.3	106303622.1
2016-2017	99185598	134555656.1	134714552.4	120535360.9

TABLE 3. MEAN ABSOLUTE PERCENTAGE ERROR OF THREE MODELS

Error Type	GM (1,1)	Improved GM (1,1)	MHGM (1,1)
MAPE (%)	0.1893	0.1905	0.1367

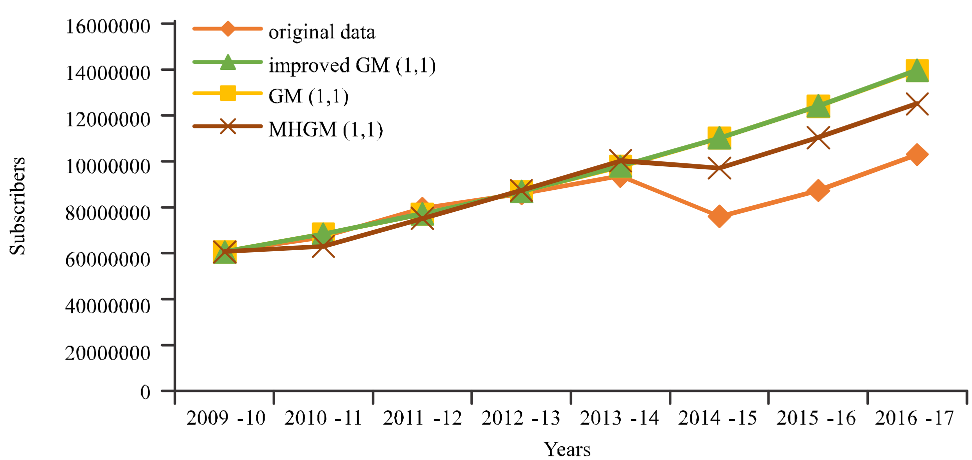

FIG. 2. GRAPHICALLY COMPARISON BETWEEN THREE MODELS AND ACTUAL DATA OF MODEL

4. CONCLUSION

In this study a new method MHGM (1,1) is given based on improved GM (1,1) and EDDGM (1,1) models to solve the forecasting problems based on small sample points. Total nine sample points are taken, four starting sample points are taken as simulate data and remaining five are used for prediction, then by comparative study of GM (1,1); improved GM (1,1) and MHGM (1,1) it is shown that MHGM (1,1) give better forecast accuracy. So; this method can be applied to other real-world growth problem when only small data is in hand.

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