Frequency Diversity Array for DOA Estimation

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RECEIVED ON 10.02.2016 ACCEPTED ON 16.08.2016

ABSTRACT

The localization of targets has been presented in this article. DOA (Direction of Arrival) is an important parameter to be determined by radar. The MLE (Maximum Likelihood Estimator) has been widely used to accurately and efficiently estimate the DOAs of multiple targets. The targets at different ranges result in a variation in amplitude of the received signals, so an MLE estimator has to operate at all ranges. For accurate results of DOA, the complex amplitudes of multiple targets should not be much different and also the prior information of Doppler and number of targets is required. In this paper, an approach is proposed which uses the classical 2D algorithm to estimate range, Doppler and number of targets and then FDA (Frequency Diversity Array) is used to focus power in a particular range. As a result, the MLE can get data from a particular range cell where all targets have almost same amplitude and thus MLE can accurately estimate the DOAs of multiple targets. Simulations and results have confirmed the effectiveness of proposed approach.

Key Words: **Radar, Direction of Arrival, Frequency Diversity Array, Maximum Likelihood Estimate, Range.**

1. INTRODUCTION

DOA estimation is an important area in radar, sonar and communication signal processing. In the last two decades, many techniques have been proposed for DOA estimation [1-5]. High resolution techniques have also been studied by radar researchers [6-10]. The monopulse technique has been used in many practical radar systems for DOA estimation [11-14]. The problem with the monopulse system is that when multiple targets are present in an azimuth cell, it gives only one direction for all the targets [15-16]. MLE methods have been presented for radar array signal processing [17-18]. Multiple targets can be correctly resolved in an azimuth cell by the using the MLE technique [19].

MLE for DOA estimation by antenna scanning is presented in [20] which accurately and efficiently estimate the DOAs of multiple targets. The targets at different ranges result in power variation of signals received after reflection from the targets. DOA estimationthrough MLE requires data from targets in same range cell (approximately same amplitude). So the MLE estimator has to operate on all ranges as the complex amplitude of multiple targets should not be varied. So for targets at different ranges, it gives errors. Also the prior information of doppler and number of targets is needed [20]. As [20], the accuracy of DOA estimation through the MLE of amplitude highly depends on power of signals reflected

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from targets in a cell. So if targets have large variation in power due to range difference, the DOA estimation has errors. When the range separation of targets is less, their DOAs are accurately estimated, but when the range separation increases, error is produced because now the amplitude of reflected signal from targets largely varies.

In this paper, an approach is presented which uses the 2D (Two Dimensional) Range/Doppler algorithm [21] to estimate range, Doppler and number of targets and then FDA [22] is used to focus the power in a particular range. We have proposed a methodology in which first the range of multiple targets is estimated then DOA estimation is performed by focusing the main beam in a particular range through FDA [23-24] so that power of the received signals in a batch remains almost same. FDA is used to concentrate beam on a particular range [22]. Thus FDA helps in reducing the DOA estimation error. Doppler and number of targets' information are used by the MLE to find the DOA of targets.

The rest of the paper is organized as follows. Section 2 describes the signal modeling and array system. Section 3 develops the proposed methodology. Section 4 shows the simulation results of the DOA estimator. Section 5 presents conclusion of this paper.

2. SIGNAL MODELING

Consider a ULA (Uniform Linear Array) with 2L sensors with l=-L,…,-1,1,…,+L. The targets are assumed to be in far-field so that plane waves are incident on the sensors. The distance between 2 sensors is d. Incident wave makes angle θ with the normal which is alternate angle to horizontal axis i.e. both are same. Fig. 1 shows the array configuration.

If H_o is the hypothesis that no target is present (noise only) and H_1 is the hypothesis that a target is present, we can write mathematically data from l^{th} sensor as:

$$H_0 : x_{1_k}^{(m)} = w_{1_k}^{(m)}$$

$$H_1 : x_{1_k}^{(m)} = \begin{cases} w_{1_k}^{(m)} \; ; k = 0,1,2,\ldots,k_0-1, k_0+N,\ldots,K-1 \\ \sum_{i=1}^{I} E_i^{(m)} \alpha_k^{(m)}(\theta_i) e^{j(2\pi f_{di} k + \psi_i)} \underbrace{e^{j\frac{l\pi \sin(\theta_{i,k}^{(m)})}{2}}}_{\text{Phase du to Sensor Position}} + w_{lk}^{(m)} \\ k = \underbrace{k_0}_{\text{Dealy Time}}, k_0+N-1 \end{cases} \tag{1}$$

Where K is 0,1,2,…, K-1; Samples, m is 0,1,2,…,M-1; Pulses received in one azimuth cell, I is 0,1,2,…,I-1; Targets present in one azimuth cell, l is -L,…, -2,-1,1,2,….,L; Sensor in Array, N is Pulse width, θ_i is DOA of i[th] target, E_i is Complex amplitude of i[th] target, $\alpha^{(m)}{}_k(\theta_i)$ is two-way antenna gain for m^{th} pulse from i[th] target, θ_B is beamwidth of mainlobe, $\theta^{(m)}{}_{i,k} = \theta_i + (\theta_B/M)$, fd_i is Doppler frequency of i^{th} target.

The distance between sensors is $d=\lambda/2$. It is assumed that the rotation is slow across different pulses, but fast within a pulse so that these assumptions can be made: $\theta^{(m)}{}_{ik} = \theta^{(m)}{}_i$, $\alpha^{(m)}{}_k(\theta_i) = \alpha^{(m)}(\theta_i)$ and the complex amplitude is assumed to be constant for all pulses within an azimuth cell i.e. $E^{(m)}{}_i = E_i$. So we can write:

$$x_{1_k}^{(m)} = \sum_{i=1}^{I} E_i \alpha^{(m)}(\theta_i) e^{j(2\pi f_{di} k + \psi_i)} e^{j\frac{l\pi \sin(\theta_i^{(m)})}{2}} \tag{2}$$

Consider the radar antenna rotates mechanically with angular velocity ω_R rad/s, the number of pulses (M) for each scan in BW (Beamwidth) is $M=\theta_B/\omega_R xPRI = \theta_B xPRF/\omega_R$ where θ_B is the BW of antenna, PRI is the pulse

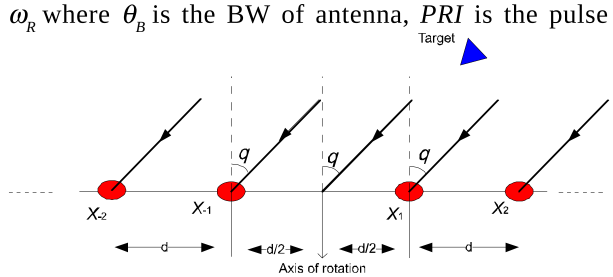

FIG. 1. THE ARRAY CONFIGURATION

repetition interval and PRF is the pulse repetition frequency of the transmitted pulses. The complete scan is divided into $c = 2\pi/\theta_B$ cells as shown in Fig. 2.

3. PROPOSED ALGORITHM

Estimation of DOA requires three steps. The proposed algorithm is presented in the coming sub-sections.

3.1 Summary of Algorithm

The overall algorithm flow is shown in Fig. 3.

To make the amplitude of received signals same, FDA concentrates beam on those targets which are at the same range and MLE can estimate the DOAs accurately. Fig.4 shows how the batch is processed.

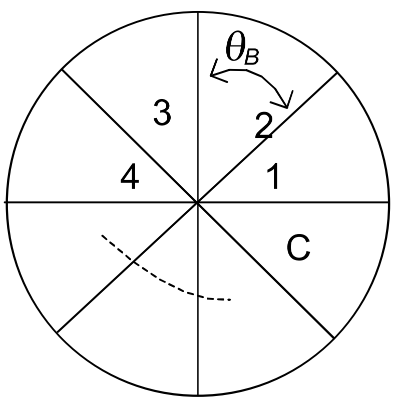

FIG. 2. AZIMUTH CELLS

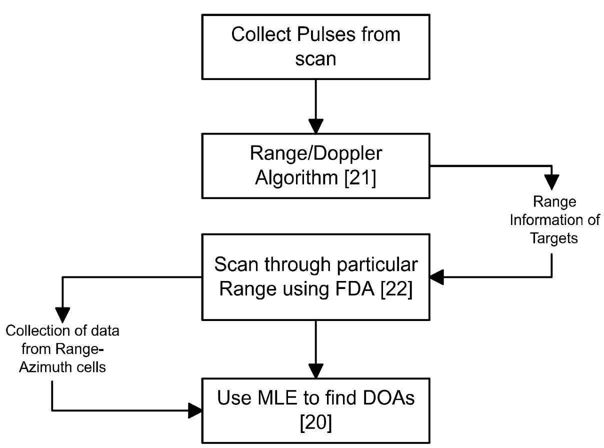

FIG. 3. ALGORITHM FLOW OF THE PROPOSED METHOD

3.2 Targets Detection (Determination of Number of Targets, Doppler, Range and Required Batch for DOA Estimation)

The array of sensors given in Fig. 1 is rotated about its axis and during the scan through azimuth cell (equal to beamwidth), all sensors' data is added.

$$X_k^{(m)} = \sum_{l=-(L-1)}^{L+1} x_{l_k}^{(m)} \tag{3}$$

The Range/Doppler Algorithm [21] is used to estimate range, Doppler and number of targets. Fig. 5 shows the steps where details are given in [21]. The range/doppler algorithm gives information about the number of targets, their range and doppler. Fig. 6 shows these results for a 10 element array and four targets

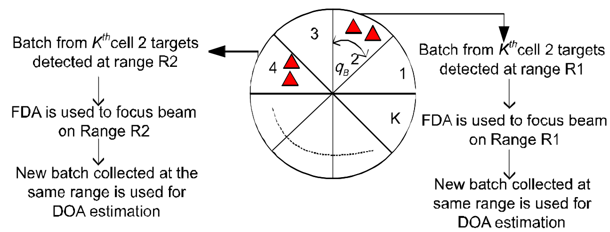

FIG. 4. BATCH COLLECTION

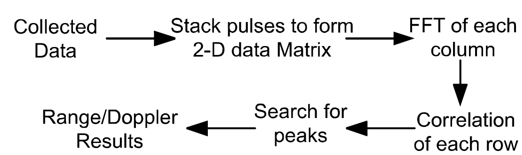

FIG. 5. RANGE/DOPPLER DETECTION ALGORITHM

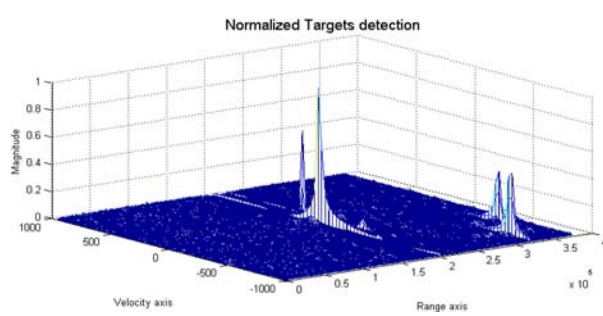

FIG. 6. DETECTION OF TARGETS

Four targets are detected with different ranges and power differences. The target parameters are given in Table 1. Target 1 and 2 are considerably closer; similarly, target 3 and 4 are close to each other. Now as 4 targets are present in different ranges so if DOA is estimated through MLE [20], error will occur due to amplitude variation. To avoid this, FDA is used to concentrate energy at specific range cell.

3.3 FDA for Focusing Beam on Particular Ranges

After the exploration of FDA [22], it is being used in many existing algorithms to improve their performance [25-26]. The FDA procedure given in [22] shows that increment in frequency gives another degree of freedom. Consider f is the fundamental frequency and Δf is the increment in frequency from one sensor to the next. So frequency of the l^{th} sensor is given by $f_l = f + l\Delta f$ where l is the sensor number. Fig. 7 shows frequency of different sensors.

If r (reference) is the range between reference and target, then the range from l^{th} sensor is given by $r_l = r - ld/2sind\theta$.

TABLE 1. TARGET PARAMETERS

Targets	Range(km), DOA(deg)
1	203, 15
2	220, 10
3	320, 72
4	350, 70

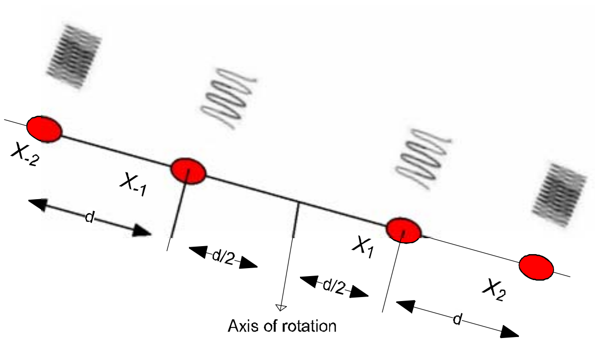

FIG. 7. FREQUENCY DIVERSITYARRAY

The phase difference between reference point and l^{th}sensor is given by:

$$\psi_n - \psi = \frac{2\pi_n r_n}{c} - \frac{2\pi f r}{c} = -\frac{2\pi_l l d/2\sin\theta}{c} - \frac{2\pi f l d/2\sin\theta}{c} + \frac{2\pi l \Delta f r}{c} \quad (4)$$

the steering vector can be written as:

$$v(\theta, r) \begin{bmatrix} e^{j\left(\frac{2\pi f l d/2\sin\theta}{c} + \frac{2\pi l \Delta f l d/2\sin\theta}{c} - \frac{2\pi l \Delta f r}{c}\right)} \\ \vdots \\ e^{j\left(\frac{2\pi f d/2\sin\theta}{c} + \frac{2\pi l \Delta f d/2\sin\theta}{c} - \frac{2\pi \Delta f r}{c}\right)} \\ \underset{\text{reference}}{1} \\ e^{-j\left(\frac{2\pi f l d/2\sin\theta}{c} + \frac{2\pi l \Delta f l d/2\sin\theta}{c} - \frac{2\pi l \Delta f r}{c}\right)} \\ \vdots \\ e^{-j\left(\frac{2\pi f l d/2\sin\theta}{c} + \frac{2\pi l \Delta f l d/2\sin\theta}{c} - \frac{2\pi l \Delta f r}{c}\right)} \end{bmatrix}$$

So the radiation also depends on range. The frequency increment and range are related by $R = c/\Delta f$ [22]. So by using Δf we can scan at different ranges. FDA gives improved localization in range [25]. Using array of 10 elements, FDA is applied to scan range cell of 190-230 km first as shown in Fig. 8. For this range, frequency increment Δf=1500 Hz is required because $\Delta f = c/R$. When the transmitted frequency is 10 GHz and frequency shift is 1500 Hz, the total shift in frequency for 10 elements becomes 15 KHz which is negligible as compared to 10 GHz. So the phase error at the receiver is also negligible due to frequency shift. Sensors are placed at a spacing $d = \lambda/2$. The power intensity is maximum at 190-230 km i.e. span of40 km. 20° beamwidth of the main lobe scans mechanically. Fig. 9 shows the magnitude of radiation pattern verses the angle directions. The mainlobe has beamwidth of 20°. The data received through the above parameters is used to find the DOAs of the targets present in the 190-230 km range. The range Vs magnitude plot is shown in Fig. 10 with the DOA estimation method and results in the next section.

As seen in Fig. 8, 2 targets were found in the 320-360 km range cell. To collect pulses from that range, now the FDA is applied to scan range cell of 320-360 km as shown in Fig. 10. Now a frequency increment of 923 Hz is applied to get mainlobe focused on the desired 320-360 km range.

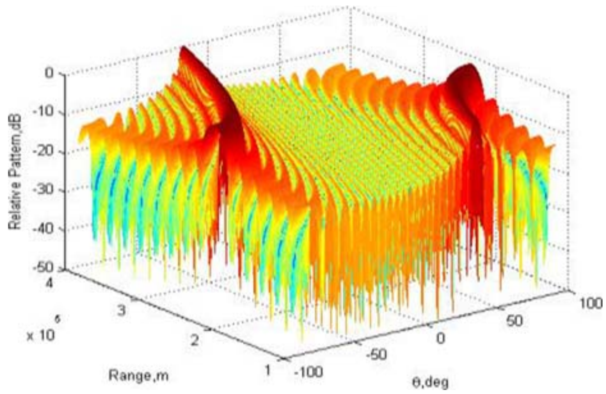

FIG. 8. RADIATION PATTERN VS RANGE AND AZIMUTH

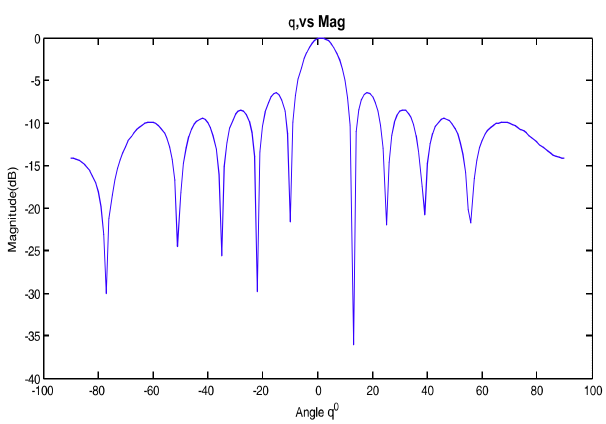

FIG. 9. RADIATION POWER VS AZIMUTH

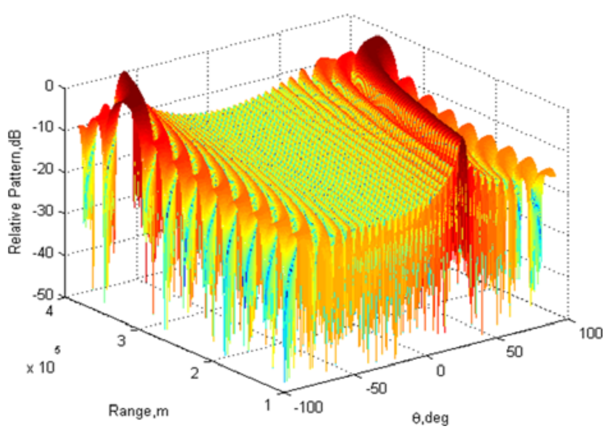

FIG. 10. Δf = 923 HZ FOR THIS RANGE CELL

3.4 DOA Estimation through MLE of Amplitude

Now the data is collected from range-azimuth cells which contain multiple targets and MLE for DOA [20] is now used to get the DOA information as shown in Fig. 11.

The sum of data from all sensors can be written as:

$$x_k^{(m)} = \sum_{i=1}^{I} E_i \alpha^{(m)}(\theta_i) e^{j(2\pi_{di}k)} \quad (5)$$

Where $\alpha^{(m)}(\theta_i)$ is the beampattern of combination of sensors

$$\alpha^{(m)}(\theta_i) = G_o e^{\left[-4\ln 2\left(\frac{\theta_i-(m+(c-1)M)\omega_R T}{\theta_B}\right)^2\right]} \quad (6)$$

Where m is pulse number, *c* is cell number, *M* is total pulses, *T* is PRI. The beampattern is Gaussian with pattern shown in Fig.12.

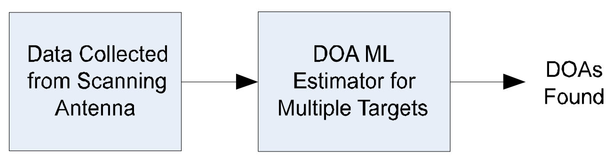

FIG. 11. DOA ESTIMATOR STAGE

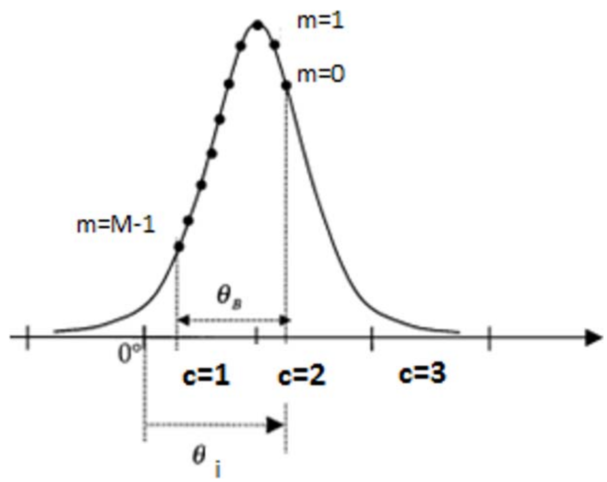

FIG. 12.AMPLITUDE VARIATION

In vector form, we can write $x=A(\theta)b+w$ where $b=[E_1, E_2, E_3 \ldots E_1]$, $A(\theta) = [\alpha_1(\theta_1)\ \alpha_2(\theta_2)\ldots\ \alpha_1(\theta_1)]$, $\alpha_i(\theta_i) = \alpha^{(m)}(\theta_i)^{j(2\pi f d i k)}$. The complex Gaussian PDF is given by [1]:

$$p_{x|b}(x|b;\theta) = \frac{1}{(\pi\sigma^2)N_{|M|}} \times e^{\left[-\frac{(x-A(\theta)b)^H M^{-1}(x-A(\theta)b)}{\sigma_w^2}\right]} \tag{7}$$

where $\boldsymbol{M}$ is the noise covariance matrix and σ^2 is variance of noise. Methods of finding $\boldsymbol{M}$ can be found in [27]. For thermal noise, it is identity $M=I$ [20]. Maximization of Equation is equivalent to minimization of the quadratic form:

$(x-A(\theta)b)^H M^{-1}(x-A(\theta)b)$

Minimization of the quadratic form is given now [28]:

$$(x-A(\theta)b)^H M^{-1}(x-A(\theta)b) = (x^H - b^H A^H) M^{-1} (x-Ab)$$
$$=(x^H M^{-1} - b^H A^H M^{-1})(x-Ab)$$
$$=x^H M^{-1} x - x^H M^{-1} Ab - b_H A_H M^{-1} x + b_H A_H M^{-1} Ab$$

The gradient w.r.t $\boldsymbol{b}$ is:

$$\partial/\partial b = -2A^H M^{-1} x + 2A^H M^{-1} Ab = 0$$
$$\Rightarrow 2A^H M^{-1} Ab = 2A^H M^{-1} x$$
$$\therefore \hat{b} = \left(A^H M^{-1} A\right)^{-1} A^H M^{-1} x$$

The DOA estimator is given by [16], Equation (9):

$$\hat{\theta} = \arg\max|_{\theta}\, Z^H M^{-1} A\left(A^H M^{-1} A\right)^{-1} A^H M^{-1} z \tag{8}$$

Equation (5) gives the DOA (θ) of target(s).

4. SIMULATION RESULTS OF DOA ESTIMATOR

In this section we present simulation results of the proposed methodology. As MLE [20] produces errors in DOA estimation when there is large range separation between the targets. Now using the modified approach, first the range and Doppler information is obtained. For simulation, we are using $L = 10$ sensors. ULA with distance between sensors, $d=\lambda/2$. According to Table 1, we have used four targets for the simulation. $r_1 = 203$ km, $\theta_1 = 15^o$, $r_2 = 220$ km, $\theta_2 = 10^o$, $r_3 = 320$ km, $\theta_3 = 72^o$, $r_4 = 350$ km, $\theta_4 = 70^o$. After applying the Range/Doppler algorithm, peaks are obtained Vs range Fig. 13 which shows the amplitude of the targets detected where we can see that the power changes with range. FDA is now used to focus the beam on the 190-230 km range to find the DOAs of first 2 targets. It can be seen in Fig.14 that the main lobe is focused in the desired range. The Range Vs magnitude plot is shown below while the theta Vs magnitude plot was shown in Fig. 9. For focusing on the range of 190-230 km, Δf is kept to 1500 Hz because $R=c/\Delta f$ as shown in Fig. 15. Now the MLE Equation is used to get information of DOA for each range azimuth cell. For 190-230 km, as shown in **F**ig. 14, targets at 15and 10^o, the received normalized beampattern is shown in Fig. 16 where the pulses of both targets are added.

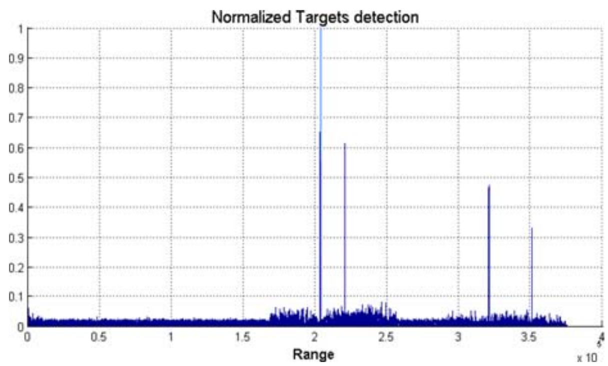

FIG. 13. THE AMPLITUDE OF TARGETS VS RANGE

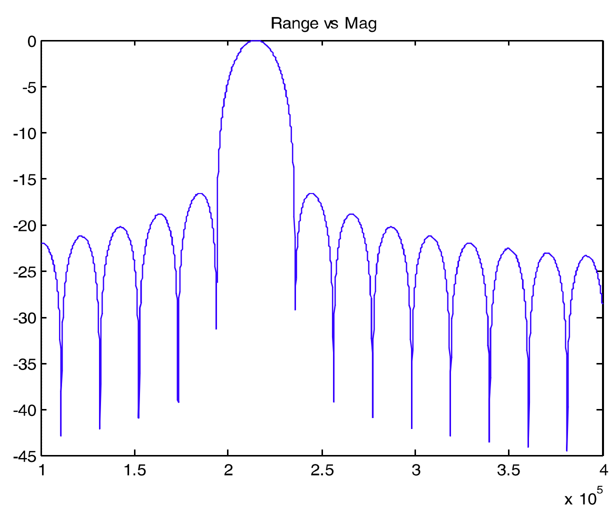

FIG. 14. MAIN LOBE IS IN THE 190-230 KM CELL

Next the DOA estimation is performed on the data received. Equation is used to take a plot of the DOA estimates of Target 1 and 2. The mesh plot below has 3 axes. The x and y-axes are for the DOA of target 1 and 2 while z-axis is representing magnitude of Equation (5).

Estimated DOAs are shown in Fig. 17. It can be seen from the maximum values in Fig.17 that the targets present at 15 and 10^o at a range of 203 and 220 km respectively are estimated correctly using proposed method. When targets are scanned separately from targets at higher range i.e. 320-360 km, the amplitude of targets did not have a lot of variation. So the proposed method estimated the DOAs correctly. Similarly, for 320-360 km, FDA is used to concentrate power of mainbeam in the desired range which is shown in Fig. 18.

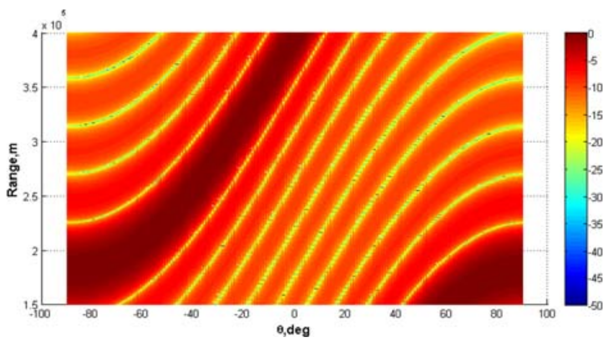

FIG. 15. RANGE VS AZIMUTH

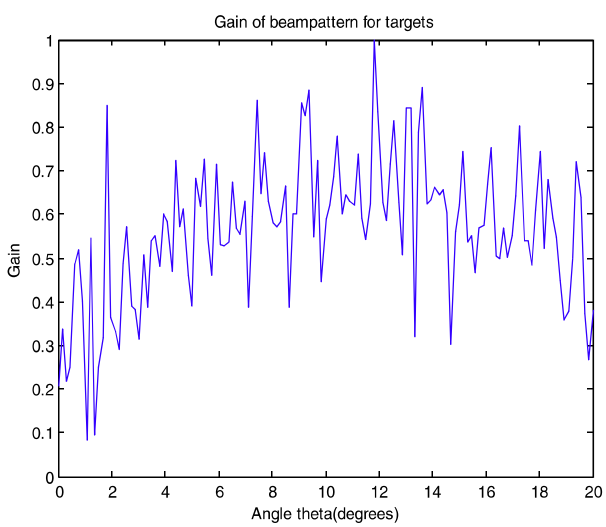

FIG. 16. BEAMPATTERN FOR MULTIPLE TARGETS IN A RANGE-AZIMUTH CELL

Two targets are present at 72 and 70^o: the received normalized beampattern is shown in Fig. 19. Fig. 18 data is collected which was focusing in the 320-360 km range for search of 2 targets i.e. target 3 and 4. The gain of antenna has the effect of pulses received from both of the targets present. DOA estimator Equation is again performed. The DOA estimates of target 3 and 4 are shown in Fig. 20. The 2 targets present in the 320-360 km range at angles of 72 and 70^o are also correctly estimated which give error using [20] without FDA. For 70 and 72^0, the radar is looking in the 4th cell of Fig. 4.

For further evaluation, the four targets are now placed at a different range and angle set. Ranges of 50, 60, 140, 150 km with DOAs of 45, 48, 58, and 50^o, respectively, are simulated. Now first FDA is used to scan targets at 50 and 60 km as shown in Fig. 21(a).

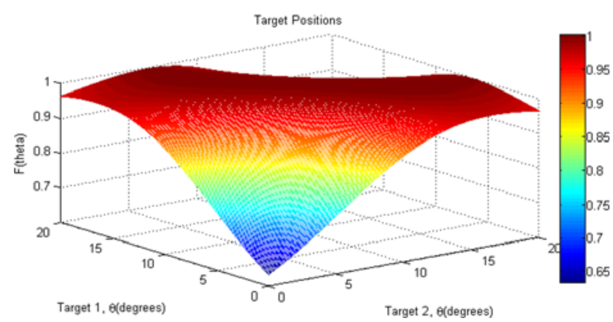

FIG. 17. DOAS ESTIMATED AT 15^O AND 10^O USING EQUATION

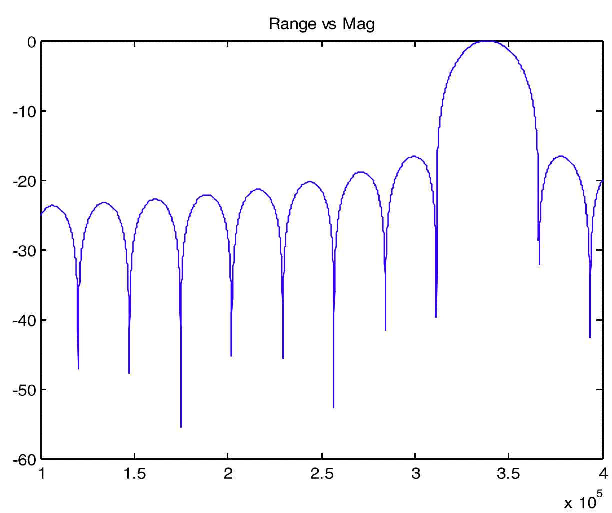

FIG. 18. RANGE VS BEAMPATTERN

The DOAs are estimated using Equation (5) as shown in Fig. 21(b). The DOAs are estimated correctly as the peak for target 1 lies at 45^o and for target 2 lies at 48^o. FDA is then used to scan targets present at 140 and 150 km as shown in Fig. 22(a). The main lobe is at the required range and the targets present at 58 and 50^o are estimated accurately using Equation (5) as shown in Fig. 22(b).

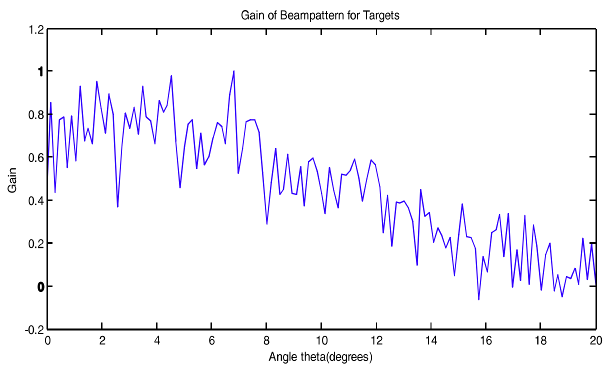

FIG. 19. BEAMPATTERN WHEN FOCUSED AT 320-360 KM RANGE

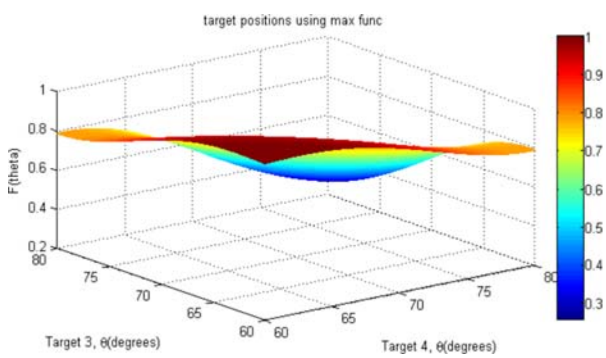

FIG. 20. TARGETS AT 72 AND 70^O, ESTIMATED USING EQUATION

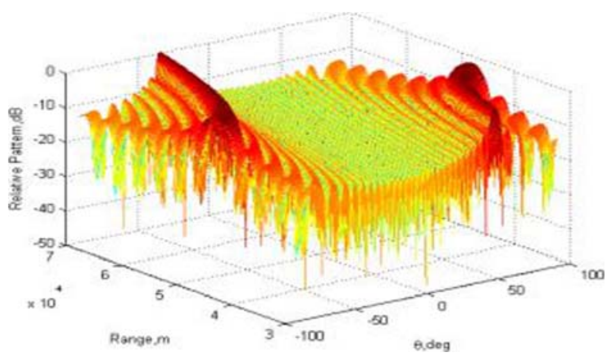

FIG. 21(a). Δf=5.4 KHZ FOR THIS RANGE CELL

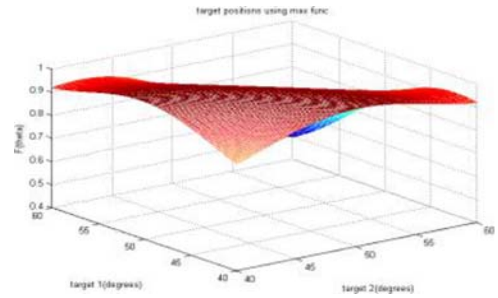

FIG. 21(b). TARGETS AT 45 AND 48^O, ESTIMATED USING EQUATION (5)

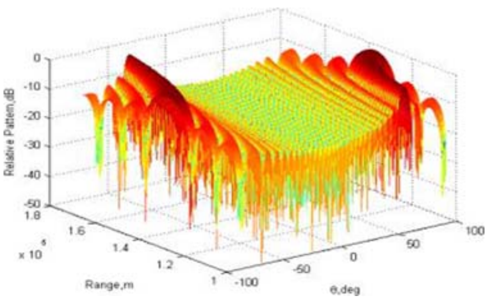

FIG. 22(a). Δf= 2.06 KHZ FOR THIS RANGE CEL

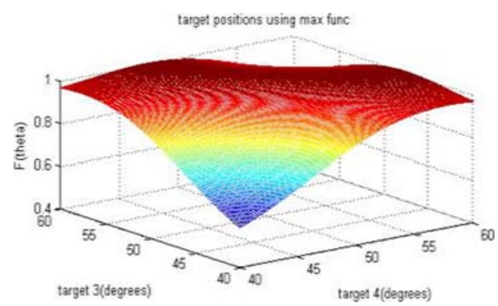

FIG. 22(b). TARGETS AT 58 AND 50^O, ESTIMATED USING EQUATION

5. CONCLUSION

In this work, we have developed a complete methodology to estimate DOA via MLE using FDA. The problems faced by MLE are highlighted and solved via FDA. Also, the parameters such as range, Doppler and number of targets are estimated. It has been shown that multiple targets at different ranges can be accurately localized using the presented method. Currently we are also working on the technique through which Elevation angle can be estimated by scanning the beam electronically through elevation angles an each azimuth. Simulation results verify the effectiveness of the approach. The frequency shift in FDA can cause phase error if the number of elements is too large or the frequency shift is very much high.

ACKNOWLEDGEMENT

The authors appreciate the support given by Mr. Asim Ajaz, Department of Electrical Engineering, College of Electrical & Mechanical Engineering, National University of Sciences & Technology, Pakistan.

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