Mehran University Research Journal of Engineering & Technology
Vol. 37, No. 2, 257-268, April 2018
p-ISSN: 0254-7821, e-ISSN: 2413-7219
DOI: 10.22581/muet1982.1802.03

On The Design and Characteristics of a Sub-Optimal Observer for Boeing-747

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RECEIVED ON 27.09.2016 ACCEPTED ON 29.05.2017

ABSTRACT

LPV (Linear Parameter Varying) system is an important class of system, as it covers many physical systems. In the existing design theory related to control system, the major part is related to linear and non-linear systems. However, the LPV system is getting prominence and hence is an attractive area of research. Control issues linked with LPV systems are an emerging area of modern research. To investigate the control of this predominant class, the idea of observer design has been carried out in this article. In this paper, an observer based on RKF (Routine KalmanFiltering) scheme and LQR (Linear Quadratic Regulator) is employed for a set of linear parameter variations. The state and gain matrices are scheduled using an interpolation method, which is linear according to each parameter and is expected to be non-linear globally. For stability of observer, bound on rate of parameter variation is imposed. For simulation purpose, a real life case study of Boeing-747 is adopted. The proposed scheme is implemented for the stated LPV system. All the associated states of the system are examined with and without observer. Results obtained in this work show better performance as manifested by errors. Error in measurement is much reduced by employing this scheme. Short-listed features are presented in this paper to comprehend the performance of observer.

Key Words: Observer, Linear Parameter Varying System, Linear Quadratic Regulator and Kalman Filter.

1. INTRODUCTION

State estimation has remained a vast area of research in control and communication system engineering. When the data signal overlaps with the noise signal, ordinary filtering schemes fail to cope the filtration and hence, state estimation comes into being. Perhaps the best known tool for state estimation of LTI (Linear Time Invariant) systems is Kalman filter [1]. However, it depends heavily on perfect knowledge of the system dynamics, information of unmeasured stochastic inputs and noisy measurement data [2]. Though, abstraction of factual system data from measurements with noise due to sensors is the key goal of the Kalman filter [3]. Towards this end, the sensor readings are used to compute the minimum mean square error estimate of the system state. There are several estimation methods which are capable for linear, LPV and nonlinear systems. Observer design for LPV systems is overgrowing field of modern research due to its wide applications.

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LPV system is an emerging class of modern research era. It covers a wide range of systems, including UAVs, turbofan engines, missiles, which are most common applications of LPV systems. There is a considerable research work on LPV observer designs [4-10] and also on LPV control using various type of controllers [11-13]. Many results of stability using gain scheduling methods have been shown [14-15], in addition Kalman type realization of LPV system is used in [16]. Frequently, in nonlinear case, the observers and/or controllers designs are based on transformation of the system's canonical form, but the design of such a transformation can be an obstruction in practical scenario. This short coming persuade the application of LPV systems scheme [17]. In some standpoints due to uncertainties (unidentified parameters and/or disruption) the design of a typical observer, merging in case of no noise to the true value of the state, is problematic [17]. An efficient way to formulate an LPV model by adding linear models obtained at various conditions [18], which will cover the variation in dynamics of a system. Apart from the above mentioned applications, another common applied area for LPV systems is aerospace problems. Perhaps the recent work on computation for LPV system's states, is application of EKF (Extended Kalman Filter) which is a nonlinear approach [19-20], but it bears the complexity of nonlinearity. In this research contribution a sub-optimal observer dependent on standard MMSE (Minimum Mean Square Error) scheme, LQR and interpolation is employed for a set of linear parameter variations. The state and gain matrices are scheduled by using an interpolation method, which is linear according to each parameter and is expected to be non linear globally. The employed observer gives appreciable results. Designed observer is implemented for a case study of Boeing 747, which is an LPV system.

The LPV model is taken from [18] and the state and gain matrices are scheduled by using an interpolation method. For stability of observer, bound on rate of parameter variation is imposed. A numerical example of LPV system is taken from [11], for which the designed observer is implemented. This research work is a linear approach towards control design of LPV system, which avoids the complexity of nonlinearity. The proposed scheme is also expected to be efficient from computational point of view. Results obtained are acceptable as presented in section 6.2.

This research contribution is coordinated as follows. Section 2 describes a generalized form of the LPV model. Section 3 presents LPV system with process and measurement noises. Section 4 proposes structure of observer for LPV systems. Section 5 presents the basic derivation involved in obtaining state matrix using RKF. Section 6 describes design state feedback controller to get gain matrix for each parameter. Section 7 shows Boeing 747 series 100/200 as a true life example and simulation results with and without employment of proposed observer. Section 8 concludes the paper with emphasis on main topics.

2. LINEAR PARAMETER VARYINGSYSTEM

In this section, a very brief discussion has been provided for LPV system; initially a generalized format followed by an LPV system subjected to Gaussian noise.

2.1 Generalized Linear Parameter Varying System

LPV systems are basically linear systems whose dynamics vary linearly with a time varying parameter, say $\theta(k)$. In normal routine, parameter itself may be time-varying but it must have specific bounds [18]. In other words, $\theta(k) \in \Theta(k)\Omega(k)$ where $\Theta(k)$ is a set with specific bounds. The parameter $\theta(k)$ may or may not be the states of the system. The LPV system presents a specific behavior at lower bound(s) of linearly varying parameter, while another distinct aptitude at upper bound. A generalized LPV system with varying parameter $\theta(k)$ can be represented as:

$$\begin{bmatrix} \dot{x}(k) \\ y(k) \end{bmatrix} = \begin{bmatrix} A(\theta(k)) & B(\theta(k)) \\ C(\theta(k)) & D(\theta(k)) \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \quad (1)$$

In the Equation (1) A(θ(k)): $R^s \rightarrow R^{nxn}$, B(θ(k)): $R^s \rightarrow R^{nxm}$, C(θ(k)): $R^s \rightarrow R^{pxn}$, and D(q(k)): $R^s \rightarrow R^{px1}$, are parameter dependent system dynamics, x(k): $R^s \rightarrow R^{nx1}$, u(k): $R^s \rightarrow R^{px1}$, y(k): $R^s \rightarrow R^{px1}$ represent state of the system, deterministic input and defined output of the above mentioned system respectively. The factor of A(θ(k)), B(θ(k)), C(θ(k)) and D(θ(k)) are polynomials of the parameter θ(k) which have specific bounds. Hence varying the system parameters θ(k), the respective system matrices also vary due to variation in its elements. The elements of these matrices are functions of that varying parameter. The following assumptions are made.

Assumption-1: Each parameter θ_i ranges between known external values $\theta_i(k) \in \left[\underline{\theta}_i, \overline{\theta}_i\right]$ and $\underline{\theta}_i \langle 0, \overline{\theta}_i \rangle 0$ [16].

Assumption-2: The rate at which each parameter $\theta_i(k)$ varies is limited by known upper and lower bounds i.e. $\dot{\theta}_i(k) \in \left[\dot{\underline{\theta}}_i, \dot{\overline{\theta}}_i\right]$ [16].

2.2 System Model with Gaussian Noise

It is hard to imagine, a system free of interruption (noise, disturbance, etc.) in practical scenario. There may be various situations where these unwanted signals may not be avoided. There may some noises like Gaussian, colored etc or it may be certain faults. The interrupted version of the above mentioned LPV system would have the following dynamics.

$$\begin{bmatrix} \dot{x}(k) \\ y(k) \end{bmatrix} = \begin{bmatrix} A(\theta(k)) & B(\theta(k)) \\ C(\theta(k)) & D(\theta(k)) \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} + \begin{bmatrix} w(k) \\ v(k) \end{bmatrix} \quad (2)$$

In the above model, v(k)k: $R^s \rightarrow R^{px1}$ andw(k)k: $R^s \rightarrow R^{nx1}$ are measurement noise and process noise vectors respectively. The two noises may be of various formats and types, depending upon different scenarios. For simplification purposes, these noises are assumed to be white Gaussian, uncorrelated, having mean equal to zero and a covariance matrices which are bounded as w(k) ≈ N(0,Q_k), and v(k) ≈ N(0,R_k) Q_k in which Q_k and R_k are matrices showing the covariance. In addition, noise process is uncombined covariance given by:

$$E\begin{bmatrix} w(k) \\ v(k) \end{bmatrix}\begin{bmatrix} w^T(s) & v^T(s) \end{bmatrix} = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}\delta_{ks} \quad (3)$$

where $Q \in R^{nxn}$, $S \in R^{nxp}$ and $R \in R^{pxp}$.

$$\delta_{ks} = \begin{Bmatrix} 1; & k = s \\ 0; & k \neq s \end{Bmatrix} \quad (4)$$

For other type of noise, the interested readers are referred to literature. Since the core objectives of the manuscript is to design and complete LPV matrices using interpolation scheme, the two noises are assumed to be simpler. In the next section proposed observer design is presented.

3. PROPOSED OBSERVER FOR LPV SYSTEMS

Consider the representation of LPV model

$$\begin{aligned} \dot{x}(k) &= A(\theta(k))x(k) + B(\theta(k))u(k) + w(k) \\ y(k) &= C(\theta(k))x(k) + v(k) \end{aligned} \quad (5)$$

with

$$A(\theta) = A_0 + \sum_{i=1}^{i=K} \theta_i A_i$$

and

$$B(\theta) = B_0 + \sum_{i=1}^{i=K} \theta_i B_i$$

where $A_0, A_1, \ldots A_K, B_0, B_1 \ldots B_K$ are known matrices and θ_i is a time varying parameter. The parameter vector θ(k) has been considered to be bounded in a Hyper-rectangle having 2^K vertices such that

$$\upsilon = \{(\upsilon_1, \upsilon_2, \ldots\ldots, \upsilon_K);\ \upsilon_i \in [\underline{\theta}_i, \overline{\theta}_i] \quad for\ \ each\ \ i \tag{6}$$

where υ_i is the i[th] vertex of hyper-rectangle. Similarly the rate of variation of this parameter θ_i is bounded to upper and lower limits and it belongs to other hyper rectangles as defined by the following set of vertices [16].

$$S = \{(\tau_1, \tau_2, \ldots\ldots, \tau_K);\ \tau_i \in [\dot{\underline{\theta}}_i, \dot{\overline{\theta}}_i] \quad for\ \ each\ \ i \tag{7}$$

In the manuscript emphasis has been made to achieve a sub optimal observer that would estimate the state of the defined LPV system, provided that the assumptions are fulfilled. In other words, a linearly parameter varying gain matrix is investigated that can estimate the state(s) sub optimally. The reader may wonder of the word sub optimal, but the fact is that generalized LPV poses infinite collection of systems upon varying parameter $\theta_i(k)$. However, the authors assume that the defined bound, variation and rate of variation of $\theta_i(k)$ would lead to finite linearly varying system denoted by Equation (1). In this case, the assumed system would lose the actual trajectory of the operation but should remain in limitations. The sub-optimal observer may be of the format

$$\dot{z}(k) = F(\theta)z(k) + G(\theta)u(k) + K(\theta)y(k) \tag{8}$$

$$x(k) = z(k) + My(k) \tag{9}$$

and can be obtained by various methods including interpolation among extreme points. These extreme points associated to the proposed observer, in order to estimate the system's state when $\theta \in \upsilon_i$. The interpolation procedure can be imagined as a linear process according to each element of parameter set.

In the design of observer, only the extreme points are considered to be found. This assumption would lead to a finite set of observer's parameters including gain matrices and the proposed Quadratic control law. In achieving the above goal, the following assumption needs to be made.

Definition-3.1: The system is assumed to be AQS (AffinelyQuardratically Stable) if there exists *K*+1 symmetric matrices $P_0, P_1, P_2, \ldots, P_K$ such that the following inequalities

$$P(\theta) = P_0 + \theta_1 P_1 + \theta_2 P_2 + \ldots\ldots + \theta_K P_K \rangle 0 \tag{10}$$

$$F(\theta, \dot{\theta}) = A(\theta)^T P(\theta) + P(\theta)A(\theta) + P(\dot{\theta}) - P_0 \langle 0 \tag{11}$$

hold $\forall \theta = [\theta 1, \theta_2, \ldots, \theta_K]^T$ from these variations, it is evident that the proposed sub optimal observer will also be of linear parameter varying format and would differ from the standard linear observer. Importantly saying, that the interpolation procedure shall be a linear process with respect to each parameter but from global solution point of view, it is expected to be product of these linear interpolations. The transition matrix $F(\theta)$, the gain matrix $K(\theta)$and input matrix $G(\theta)$ are determined by interpolation and hence, $F(\theta)$, $G(\theta)$ and $K(\theta)$ are defined by a polytope having $\Re^{nxn}$, $\Re^{nxm}$ and $\Re^{nxp}$ dimensions respectively. In other words:

$$F = F_0 + F_1, \ldots, F_{2K-1} \tag{12}$$

$$K + K_0 + K_1, \ldots, K2K\text{-}1 \tag{13}$$

$$G = G_0 + G_1, \ldots, G_{2K-1} \tag{14}$$

Every F_i of F, K_i of K and G_i of G, relates to a specific vertex of υ.The relationship among these vertices can be explained in the following paragraph.

Let b_j for j={0,1,2,…,K-1} be the binary representation of index *i*. In such a case, the polytope vertices associated to F_i, K_i and G_i are $(\tilde{\theta}_0, \tilde{\theta}_1, \tilde{\theta}_2, \ldots\ldots, \tilde{\theta}_{K-1})$ where $\tilde{\theta}_j$ shows extreme values such that :

$$\tilde{\theta}_j = \begin{cases} \overline{\theta}_j; & if\ b_j = 1 \\ \underline{\theta}_j; & if\ b_j = 0 \end{cases} \tag{15}$$

Thus, the interpolation scheme matrices are:

$$F(q), m_0(q)F_0 + \ldots + m_{2k-1}(q)F_{2K-1} \tag{16}$$

$$K(q) = m_0(q)K_0 + \ldots + m_{2k-1}(q)K_{2K-1} \quad (17)$$

$$G(q) = m_0(q)G_0 + \ldots + m_{2k-1}(q)G_{2K-1} \quad (18)$$

Where $\mu_i(\theta)\forall_i = \{0,1,2,\ldots,2^K-1\}$ are nonlinear functions. These functions are bounded as:

$$m_0(q) + m_1(q) + \ldots + m_{2K-1}(q) = 1 \quad (19)$$

Where $0 < \mu_i(\theta) < 1$ and each interpolation function is given by:

$$\mu_i(\theta) = \prod_{j=0 \& i=\{b_0,b_1,\ldots\ldots,b_{k-1}\}}^{K-1} \frac{\alpha_j \theta_j + \beta_j}{\underline{\theta}_j - \bar{\theta}_j} \quad (20)$$

$$\alpha_j = \begin{cases} 1; & where\ b_j = 0 \\ -1; & where\ b_j = 1 \end{cases} \quad (21)$$

$$\beta_j = \begin{cases} -\bar{\theta}_j; & where\ b_j = 0 \\ \underline{\theta}_j; & where\ b_j = 1 \end{cases} \quad (22)$$

Matrices F_i, K_i, andare calculated for each element of parameter vector are interpolated as follow:

Transition Matrix

$$F(\theta) = \sum_{i=0}^{2^K-1} (\prod_{j=0}^{K-1} \theta_j^{bj}) F_i = F_0 + \sum_i \theta_i F_i + \sum_{i \neq j} \theta_i \theta_j F_{i+j+1} + \ldots\ldots + \prod_i \theta_i F_{2^K-1}$$

Gain Matrix

$$K(\theta) = \sum_{i=0}^{2^K-1} (\prod_{j=0}^{K-1} \theta_j^{bj}) K_i = K_0 + \sum_i \theta_i K_i + \sum_{i \neq j} \theta_i \theta_j K_{i+j+1} + \ldots\ldots + \prod_i \theta_i K_{2^K-1}$$

Input Matrix

$$G(\theta) = \sum_{i=0}^{2^K-1} (\prod_{j=0}^{K-1} \theta_j^{bj}) G_i = G_0 + \sum_i \theta_i G_i + \sum_{i \neq j} \theta_i \theta_j G_{i+j+1} + \ldots\ldots + \prod_i \theta_i G_{2^K-1}$$

With the above format of interpolated strategy, it is believed that the task of observer design could be reduced to finding the matrices $F^i, F^i, G^i \forall^i = [1,2,\ldots,2^K-1]$ in order to sub optimally estimate the state of an affinely LPV system. In other words, a total of $3x2^K$ matrices need to be computed to carry out the estimation process.

4. STEPS INVOLVED IN SUBOPTIMAL OBSERVER

Continuous time Kalman filter for LTI systems have been shown in numerous literature such as [2,21]. In this section, the existing Kalman filter (discrete time) is adjusted for the above mentioned LPV system model with noise. Kalman filter, predicts state of the system, using a previous output and input data samples.

$$\dot{x}(k) = A(\theta(k))x(k) + B(\theta(k))u(k) + w(k) \quad (23)$$

$$y(k) = C(q(k))x(k) + v(k)\ y(k) \quad (24)$$

where $A(\theta(k)):R^s \rightarrow R^{nxn}$ system's transition matrix: dependent on parameter, $B(\theta(k)):R^s \rightarrow R^{nxm}$ is input matrix: dependent on parameter, $C(\theta(k)):R^s \rightarrow R^{pxn}$ is output matrix: dependent on parameter. $x(k):R^s \rightarrow R^{nx1}$, $u(k):R^s \rightarrow R^{px1}$, $y(k):R^s \rightarrow R^{px1}$ represent state of a system, deterministic input and output of a system respectively. $w(k):R^s \rightarrow R^{nx1}$ is noise in the process and $v(k):R^s \rightarrow R^{px1}$ is noise in the measurement. The scheme is initialized as follow:

4.1 Initiation Step

The following assumptions are made for initiation purpose

$$X(\theta) = 0 \quad (25)$$

$$F(\theta) = A(\theta) \quad (26)$$

$$F_\theta(0) = A_\theta(0) \quad (27)$$

4.2 First Estimation step

In this step of prediction, states or state of a system are/ is predicted as shown

$$\dot{x}(k) = A(\theta(k))x(k) + B(\theta(k))u(k) \quad (28)$$

The covariance matrix of the corresponding error will be:

$$\dot{\mathrm{P}} = \mathrm{E}[\dot{e}(k)\dot{e}^T(k)] \quad (29)$$

$$\dot{\mathrm{P}}(k) = A(\theta(k))\mathrm{P}(k) + \mathrm{P}(k)[A(\theta(k))]^T + \mathrm{Q} - \mathrm{P}(k)C^T(\theta(k))R^{-1}C(\theta(k))\mathrm{P}(k)$$

The change in the actual and predicted outputs known as innovation is given as:

$$\mathrm{I}nn = y_a - \widehat{y} \quad (30)$$

where y_a is actual output and $\widehat{y}$ is estimated output.

4.3 Observer Gain

The sub optimal value of modified observer gain matrix is calculated as:

$$\mathrm{K(q(k))} = \mathrm{P}_{pred}[\mathrm{C(q(k))}][\mathrm{R}]^{-1} \quad (31)$$

It can be seen, as expected that observer gain elements depend on linear parameter varying element θ(k). Gain is calculated for each element of parameter vector linearly.

4.4 Updated Estimation

The estimation achieved in step 4.2 can be updated using the schedule gain, computed in step 4.3

$$\dot{x}_{up}(k) = \dot{x}_{pred}(k) + [K(\theta(k))]\mathrm{I}nn \quad (32)$$

where K(q(k)) is sub optimal observer gain matrix. Update error covariance is:

$$\dot{\mathrm{P}}_{up}(k) = (\mathrm{I}_{n\times n} - [K(\theta(k))][C(\theta(k))])\dot{\mathrm{P}}_{pred} \quad (33)$$

Equations (29-33) describe the basic steps involved in the design of suboptimal observer. It is important to say that various matrices involved in the design procedure are computed using interpolation scheme, discussed earlier. This scheme makes the computation cumbersome and computationally expensive. Gaussian theory implies that the innovation term is uncorrelated with x(k) and u(k). The following standard Gaussian theory assumptions can be made to ease the calculation process.

$$\mathrm{E}[\mathrm{x(k)Inn^T}] = 0_{nxp} \quad (34)$$

$$\mathrm{E}[\mathrm{u(k)Inn^T}] = 0_{(n+sms)xp} \quad (35)$$

Parameter θ(k) is also uncorrelated with x(k). For an affine LPV system, the use exhibits the following properties;

$$\mathrm{E}[\mathrm{x(k)q(k)^T}] = 0_{nxsn} \quad (36)$$

$$\mathrm{E}[\mathrm{u(k)q(k)^T}] = 0_{nxsm} \quad (37)$$

On the other hand, $\mathrm{E}[\mathrm{x(k)v(k)^T}] = 0_{nxp}$, which results in

$$\mathrm{E}[\hat{x}(k)x(k)^T]C^T = 0_{n\times p} \quad (38)$$

It shows that either $\mathrm{E}[\hat{x}(k)x(k)^T]$ is rank deficient matrix with its rows lying in the orthogonal complement of C, or $\widehat{x}(k)$ and x(k) are uncorrelated. For cross covariance matrix to be rank deficient x(k) and $\widehat{x}(k)$ must be depended vectors, which is not possible. As a result, and x(k) are uncorrelated.

5. GAIN MATRIX USING STATE FEEDBACK CONTROLLER DESIGN

It has been discussed that variation of P(·) causes variation in the matrices, and hence the issue of stability arises. To overcome this issue, a robust control law would be needed. An LQR controller scheme, based on state feedback control is employed in this paper. The state feedback gain elements are organized to achieve very fast eigen values (considerably faster than the average rate operating point's changes with a defined bound) by constructing the ARE (Algebraic Riccati Equation) with proper matrices having definite weighting. For the LPV system, given uniform controllability and observability of important pairs of matrices, the gain of observer stabilizes the plant, in spite of fast fluctuations in operating point by utilizing the result of a RDE (Riccati Differential Equation) that is time-varying [14]. Perhaps the major pre-eminence of LQR is that it is well applicable for time varying system dynamics. In this manuscript, the LQR parameters are used to minimize the undesired alteration and for decreasing the cost, where cost function is the difference of actual measurement from the desired measurement.

Consider the uninterrupted LPV system

$$\dot{x}(k) = A(\theta(k))x(k) + B(\theta(k))u(k) \tag{39}$$

A(θ(.)) and B(θ(.)) can be stabilized. An in standard scheme state feedback control, u*k) = Fx(k) cab be employed to stabilize the unstable system based on the following two specifications

(1) Transient response specification

(2) Magnitude constraints on x(k) and u(k)

Controller Setting:

Choose $Q: R^s \rightarrow R^{nxn}$ and $R: R^s \rightarrow R^{nxm}$ so that $Q = MM^T$ with $(A(\theta(k)),M)$ detectable and $R = R^T > 0$. Solving the ARE

$$PA(\theta(k)) + A(\theta(k))^T P + Q - P\,B(\theta(k))\,R^{-1}\,B(\theta(k))^T P = 0$$

The above equation is solved for **P**, which is used in calculation of feedback gain matrix $F: R^s \rightarrow R^{nx1}$ as:

$$F(\theta(k)) = -R^{-1}\,B(\theta(k))^T P \tag{40}$$

States are fed back to the system with gains computed in feedback gain matrix F(θ(k)). Simulating the initial response of:

$$\dot{x}(k) = [A(\theta(k)) + B(\theta(k)\mathrm{F}]x(k) \tag{41}$$

For different initial conditions of the specification of transient response and the constraints on magnitude, the response of the system is checked. Typically Q and R are taken as:

$$Q = \begin{bmatrix} q_{11} & 0 & 0 & \cdots & 0 \\ 0 & q_{22} & 0 & \cdots & 0 \\ 0 & 0 & q_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & q_{nn} \end{bmatrix} \tag{42}$$

$$R = \begin{bmatrix} r_{11} & 0 & 0 & \cdots & 0 \\ 0 & r_{22} & 0 & \cdots & 0 \\ 0 & 0 & r_{33} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & r_{nm} \end{bmatrix} \tag{43}$$

Order of 'Q' matrix depends on order of A(θ(k)) and order of 'R' matrix depends on order of B(θ(k)).

6. NUMERICAL SIMULATION RESULTS

The model of aircraft adopted in this paper is Boeing 747 series 100/200. This case study has been chosen since it has wide array of features (Flap with leading and trailing edges, spoilers, different surfaces for control, Jet engines with four fans) which makes it good representative for most of nowadays flying airplanes. In other words it can be imagine a better test bed to verify the adaptability of LPV design and modeling techniques [18]. For B747-100/200 aircraft, the system variables have got the factors as polynomial of parameter θ(k) [11]. The parameter is also a polynomial of total air speed (v_{tas}) and angle of attack (α), where (α), and (v_{tas}) have defined bounds i.e.

- (α) ranges in [-2, 8]æ%.
- (v_{tas}) ranges in [150, 250] m/s.

6.1 Boeing-747 LPV System Model

The typical model for the aircraft mentioned above is as follow:

$$\begin{bmatrix} \dot{x}(k) \\ y(k) \end{bmatrix} = \begin{bmatrix} A(\theta(k)) & B(\theta(k)) \\ C(\theta(k)) & D(\theta(k)) \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \end{bmatrix} \tag{44}$$

In the above model the system variables are $A(\theta) \rightarrow R^{nxn}$, $B(\theta) \rightarrow R^{nxm}$, $C(\theta) \rightarrow R^{pxm}$ and $D(\theta) \rightarrow R^{nxq}$ which depends on parameter θ(k)affinely as given by:

$$\left. \begin{aligned} A(\theta) &= A_0 + \sum_{i=1}^{7} (A_i \theta_i) \\ B(\theta) &= B_0 + \sum_{i=1}^{7} (B_i \theta_i) \\ C(\theta) &= [0 \ \ I_3] \\ D(\theta) &= 0 \end{aligned} \right\} \tag{45}$$

Initial states value $x(0)$ has taken to zero. The parameter (θ) is given by

$$[\theta_1 \ \ \theta_2 \ \ \theta_3 \ \ \theta_4 \ \ \theta_5 \ \ \theta_6 \ \ \theta_7] = [\bar{\alpha} \ \ \bar{v}_{tas} \ \ \bar{v}_{tas}\bar{\alpha} \ \ \bar{v}^2_{tas} \ \ \bar{v}^2_{tas}\bar{\alpha} \ \ \bar{v}^3_{tas} \ \ \bar{v}^4_{tas}]$$

D(θ) is taken zero for simulation purpose.

where

$$\left. \begin{aligned} \bar{\alpha} &= \alpha - \alpha_{|trim|} \\ \bar{v}_{tas} &= v_{tas} - v_{tas|trim|} \end{aligned} \right\} \tag{46}$$

Trim values of the system's states are:

$$[\alpha_{|trim|} \ \ q_{|trim|} \ \ v_{tas|trim|} \ \ \phi_{|trim|}] = [1.05^0 \ \ 0^0/s \ \ 227.02m/s \ \ 1.05^0] \tag{47}$$

Trim values of the system's inputs are:

$$[\delta_{e|trim|} \ \ \delta_{s|trim|} \ \ T_{n|trim|}] = [0.163^0 \ \ 0.590^0 \ \ 42{,}291N] \tag{48}$$

Unknown matrices for the system are calculated using interpolation of matrices found in the previous step as shown

$$F(\theta) = \sum_{i=0}^{2^K-1} (\prod_{j=0}^{K-1} \theta_j^{bj}) F_i = F_0 + \sum_i \theta_i F_i + \sum_{i \neq j} \theta_i \theta_j F_{i+j+1} + \ldots\ldots + \prod_i \theta_i F_{2^K-1} \tag{49}$$

Expanding the above expression for the mentioned system:

$$\begin{aligned} F(\theta) &= F_0 + \theta_1 F_1 + \theta_2 F_2 + \theta_3 F_3 + \theta_4 F_4 + \theta_5 F_5 + \\ &\theta_6 F_6 + \theta_7 F_7 + \theta_1\theta_2 F_4 + \theta_1\theta_3 F_5 + \theta_1\theta_4 F_6 + \\ &\theta_1\theta_5 F_7 + \theta_2\theta_3 F_6 + \theta_2\theta_1 F_4 + \theta_3\theta_1 F_5 + \theta_4\theta_1 F_6 + \\ &\theta_4\theta_2 F_7 + \theta_5\theta_1 F_7 + \theta_1\theta_2\theta_3\theta_4\theta_5\theta_6\theta_7 F_8 \end{aligned} \tag{50}$$

Similarly

$$G(\theta) = \sum_{i=0}^{2^K-1} (\prod_{j=0}^{K-1} \theta_j^{bj}) G_i = G_0 + \sum_i \theta_i G_i + \sum_{i \neq j} \theta_i \theta_j G_{i+j+1} + \ldots\ldots + \prod_i \theta_i G_{2^K-1} \tag{51}$$

For this system it will be:

$$\begin{aligned} G(\theta) &= G_0 + \theta_1 G_1 + \theta_2 G_2 + \theta_3 G_3 + \theta_4 G_4 + \theta_5 G_5 + \\ &\theta_6 G_6 + \theta_7 G_7 + \theta_1\theta_2 G_4 + \theta_1\theta_3 G_5 + \theta_1\theta_4 G_6 + \theta_1\theta_5 G_7 + \\ &\theta_2\theta_3 G_6 + \theta_2\theta_1 G_4 + \theta_3\theta_1 G_5 + \theta_4\theta_1 G_6 + \theta_4\theta_2 G_7 + \\ &\theta_5\theta_1 G_7 + \theta_1\theta_2\theta_3\theta_4\theta_5\theta_6\theta_7 G_8 \end{aligned} \tag{52}$$

and

$$K(\theta) = \sum_{i=0}^{2^K-1} (\prod_{j=0}^{K-1} \theta_j^{bj}) K_i = K_0 + \sum_i \theta_i K_i + \sum_{i \neq j} \theta_i \theta_j K_{i+j+1} + \ldots\ldots + \prod_i \theta_i K_{2^K-1} \tag{53}$$

Now for this system

$$\begin{aligned} K(\theta) &= K_0 + \theta_1 K_1 + \theta_2 K_2 + \theta_3 K_3 + \theta_4 K_4 + \\ &\theta_5 K_5 + \theta_6 K_6 + \theta_7 K_7 + \theta_1\theta_2 K_4 + \theta_1\theta_3 K_5 + \theta_1\theta_4 K_6 + \\ &\theta_1\theta_5 K_7 + \theta_2\theta_3 K_6 + \theta_2\theta_1 K_4 + \theta_3\theta_1 K_5 + \theta_4\theta_1 K_6 + \\ &\theta_4\theta_2 K_7 + \theta_5\theta_1 K_7 + \theta_1\theta_2\theta_3\theta_4\theta_5\theta_6\theta_7 K_8 \end{aligned} \tag{54}$$

Resulting estimated LPV model with the above interpolated dynamics are simulated with and without the designed observer. Acceptable results obtained are shown in Section 6.2.

6.2 Simulation Results

This section describes all the simulation results associated with Boeing 747 series 100/200. The LPV system as discussed in the previous section is checked with and without implementation of LQR controller. The various outputs including actual measurements and estimated results are presented to express the performance of proposed controller.

6.2.1 Response of LPV System without LQR

The system is observed for various input signals however a unit step response is shown in this paper to elaborate the modified observer. The step response results are shown in Fig. 1.

It can be seen from the figure that the system is not converging. In other words, the output of the system is exceeding the limits for unit step input. For stability purpose, LQR is employed.

6.2.2 Response of LPV System with LQR

Implementing LQR controller the system is observed for step input. The result is shown in Fig. 2.

It can be seen from Fig. 2 that applying LQR controller, the system can be stabilized. The output is bounded for bounded step input. Now this system is subjected to random interruption (Gaussian noise), afterwards observer is employed, results are shown.

6.2.3 Implementation of Sub-Optimal Observer

A sub-optimal observer has been implemented for the above mentioned scenario in the subsequent section. The observed states using sub-optimal observer are described in various Fig. 3.

Fig. 3 shows the results of the designed observer, related to LPV system state (angle of attack). The continuous

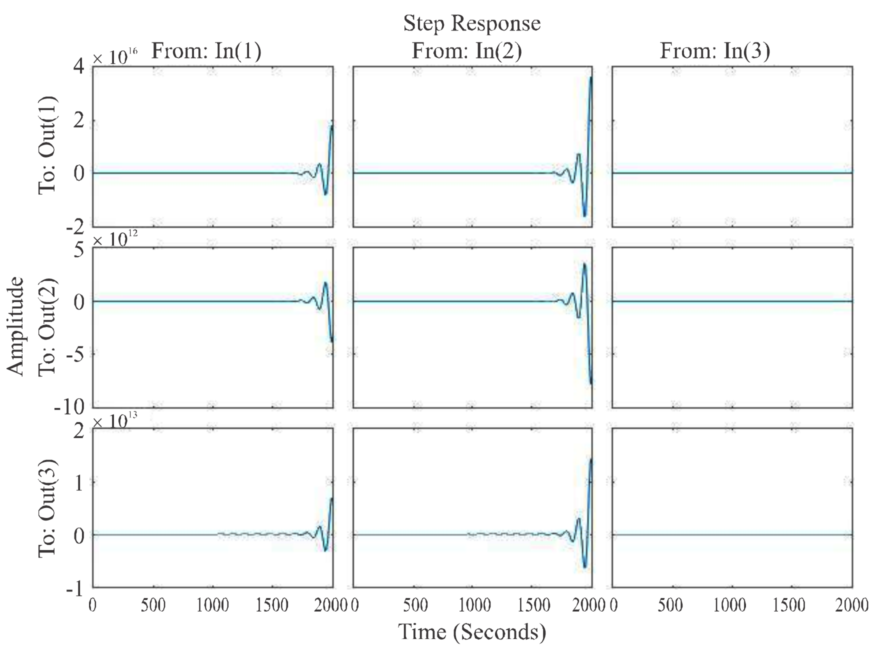

FIG. 1. UNSTABLE STEP RESPONSE (WITHOUT CONTROLLER)

line represents the true values of angle of attack, dotted line depicts measurements and dashed line is meant for the observed angle of attack. It is clear that the measured state shows more deviation from the true state. The Fig. 3 also shows that the observed state one is closer to the true state. Hence, the observed results outperform the measured results.

Fig. 4 shows the associated results of pitch rate. Evidently, it can be seen that the estimated pitch rate by the observer surpasses the measured pitch rate. It tracks the actual results and avoids the effect of interruptions.

Finally, the performance of modified observer is tested in view of the state, speed of air characteristics. In Fig. 5, the observer provides better results for this parameter too.

To evaluate the performance of the proposed scheme percentage error is computed in both measurement and observed output. Table 1 shows the percentage error.

It is clear from table 1 that the percentage error has been much reduced by employing the proposed observer, which manifests the better performance of this scheme.

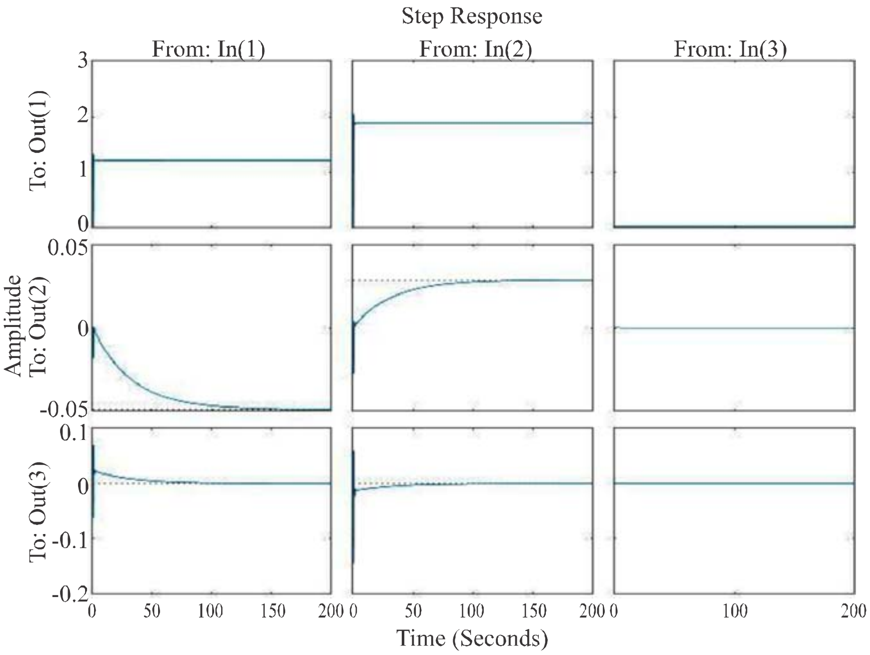

FIG.2. STABLE STEP RESPONSE (WITH CONTROLLER)

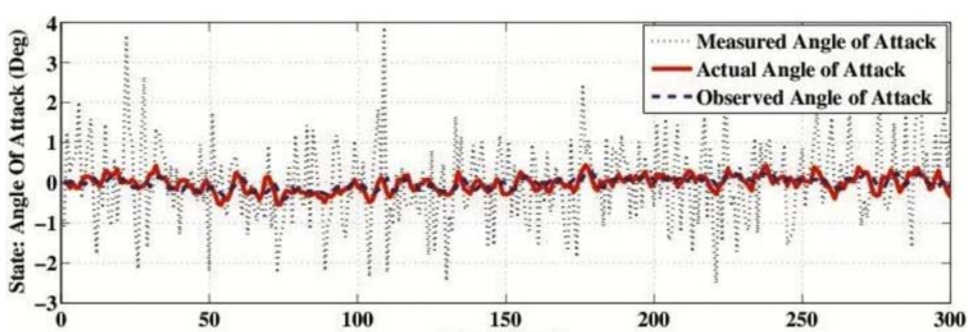

FIG. 3. ANGLE OF ATTACK

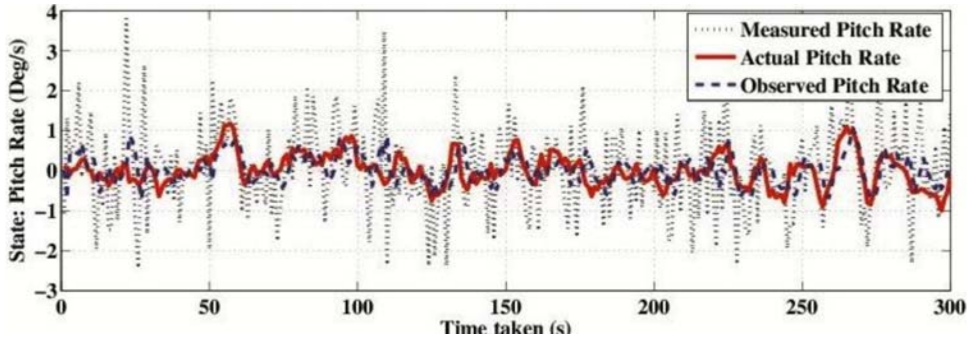

FIG. 4. PITCH RATE

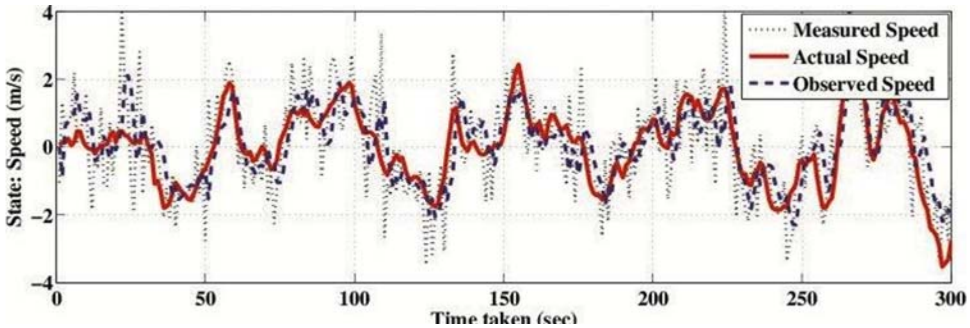

FIG. 5. SPEED OF AIR

TABLE 1. PERCENTAGE ERROR COMPARISON

No.	State	Percentage Error in Measured Parameter (%)	Percentage Error in Observed Parameter (%)
1.	Angle of attack	58.39	21.17
2.	Pitch rate	56.55	19.03
3.	Speed of air	54.8	13.98

ACKNOWLEDGEMENT

The authors are grateful to anonymous Internal and External Reviewers for this useful comments, and University of Engineering & Technology, Peshawar, Pakistan. Special thanks goes to Prof. Dr. Halim Alwi, for his sincere assistance and motivations.

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