The Use of Fractional Fourier Transform for the Extraction of Overlapped Harmonic Chirp Signals

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ABSTRACT

In ultrasound harmonic imaging, high bandwidth is essential to provide good axial resolution at the receiver. However, increasing the bandwidth will cause overlapping amongst the nonlinear harmonic components. Conventional frequency domain filtering approach normally fails to extract the desired harmonic component under overlapped harmonic condition. The purpose of this work is to use broadband linear chirp signals for better SNR (Signal to Noise Ratio) and axial resolution. Also on the receiving side, the FrFT (Fractional Fourier Transform) is used as a filtering tool to extract the overlapped second harmonic component from the nonlinear received signal. Pulse compression of the second harmonic chirp signal is also performed using the harmonic matched filter to restore axial resolution. Comparing the results of FrFT pre-filtered with unfiltered compressed signals; it is found that up to a 35.5dB RSLL (Reduction in Range Side Lobe Level) is realisable leading to an improved detection and image resolution.

Key Words: **Linear Frequency Modulation, Ultrasound, Harmonic Imaging, Harmonic Matched Filter, Fractional Fourier Transform.**

1. INTRODUCTION

THI (Tissue Harmonic Imaging) and ultrasound contrast imaging are well known imaging techniques in clinical practice. In these techniques, images are formed using the nonlinear second harmonic component of the received signal. The image produced by these techniques give better image quality and reduced side lobe artifacts when compared with conventional ultrasound imaging technique [1-4].

The main issue in the nonlinear harmonic imaging is the spectral overlapping amongst the nonlinear harmonic components. This happens when broadband signals are used as excitation. Conventional Fourier domain band-pass filtering techniques are normally unable to extract the desired harmonic under the overlapped harmonic condition [5-7].

In order to avoid spectral overlapping, narrow bandwidth signals can be used for excitation. However, narrow bandwidth signals will reduce the image resolution at the receiver. A multiple excitation technique known as PI (Pulse Inversion) can be used to extract the second harmonic

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under the overlapped harmonic condition. The main advantage of the PI technique is the enhancement of SNR of the second order harmonic component. However, PI is susceptible to motion artifact and reduced system frame rate [7-8].

Linear frequency modulation or chirp signals are now widely used in medical ultrasound systems. Chirp signals are broadband, long in duration and provide potentially high image resolution and SNR at the receiver. Also it requires a single transmission of excitation; therefore, it will be less susceptible to motion artifacts arising from tissue motion [9-11].

The aim of this paper is to use broadband chirp coded signals as an excitation and on the receiving side, nonlinear signals in which harmonic components are significantly overlapped are processed using the FrFT. Results are presented which show the extraction of overlapped second harmonic component using the FrFT. Comparisons were also made between unfiltered and FrFT pre-filtered compressed harmonic chirp signals using harmonic matched filters.

2. THE FrFT

The FrFT was originally proposed by Namias [12] and its current modified form was presented by McBride and McBride, et. al. [13]. The FrFT is a generalisation of the conventional Fourier transform and provides an important tool for the analysis and processing of linear chirp signals, Capus, et. al. [14]. Conventional Fourier transforms uses sinusoidal basis function for signal decomposition whereas FrFT define the input signal in terms of an orthonormal basis function formed by linear chirps. Therefore, the FrFT is more suitable for the analysis of linear chirp signals.

The standard Fourier transform can be expressed by:

$$X(f) = \int_{-\infty}^{\infty} B(f,t)x(t)dt \tag{1}$$

In Equation (1), B(f,t) is the Fourier transform kernel and its equals to:

$$B(f,t) = exp(-j2\pi ft) \tag{2}$$

Similarly the expression of the modified FrFT is defined as [15]:

$$F^{\alpha} f(x) = \int_{-\infty}^{\infty} B_{\alpha}(x,y)f(x)dx \tag{3}$$

In Equation (3), $B_\alpha(x,y)$ is the transform kernel of the FrFT and its equal to:

$$B_{\alpha(x,y)} = A_{\varphi} exp[j\pi(x^2 \cot\varphi - 2xy\csc\varphi + y^2 \cot\varphi)] \tag{4}$$

Where

$$A_{\varphi} = \left(2\pi\left|\sin\varphi\right|\right)^{\frac{1}{2}} \exp\left[\frac{-j\pi \, \mathrm{sgn}\left(\sin\varphi\right)}{4} + j\frac{\varphi}{2}\right] \tag{5}$$

$$\varphi = \frac{\alpha\pi}{2} \tag{6}$$

α is the transform order and their values are in the range of $\{0.0 \leq \alpha \leq 1.0\}$.

Putting the values of B_α and A_φ in Equation (3) we get [14]:

$$F^{\alpha} f(x) = \frac{\exp\left[-j\frac{1}{4}\pi\hat{\varphi} - \frac{1}{2}\varphi\right]}{\left(2\pi\left|\sin\varphi\right|\right)^{\frac{1}{2}}} \exp\left(\frac{1}{2}jy^2 \cot\varphi\right) \times \int_{-\infty}^{\infty} \exp\left(-\frac{jxy}{\sin\varphi} + \frac{1}{2}jx^2 \cot\varphi\right) f(x)dx \tag{7}$$

Where

$$\hat{\varphi} = \mathrm{sgn}\left(\sin\varphi\right) \tag{8}$$

In order to understand Equation (7), it can be divided into four steps [16].

Step-1: Multiplication of input signal by linear chirp.

Step-2: Scaled Fourier transform.

Step-3: Chirp multiplication in the Fourier transform domain.

Step-4: Complex scaling.

Consider the different values of α:

When $\alpha = 0$ or 4, $\varphi = 0$ or 2π: the transform kernel becomes $B_0(x,y) = \delta(y-x)$ which is the identity operator and gives simple time domain signal: $F^0f(x)$ or $F^4f(x) = f(x) = If(x)$.

When $\alpha=1$, $\varphi=\pi/2$: the transform kernel becomes $B_1(x,y) = A_\varphi \exp[j\pi(-2xy)]$ which is the Fourier operator and gives frequency domain signal: $F^1f(x) = F(X) = Ff(x)$.

When $\alpha=2, \varphi=\pi$: the transform kernel becomes $B_2 = (x,y) = \delta(y+x)$ which is the Reflection operator and gives reverse time domain signal: $F^2f(x) = f(-x) = F\{Ff(x)\}$.

When $\alpha=3$, $\varphi=3\pi/2$: the transform kernel becomes $B_3(x,y) = A_\varphi \exp[j\pi(2xy)]$ which is the Inverse Fourier operator and gives the Fourier transform of reverse time domain signal: $F^3f(x) = F(-X) = F\{F^2f(x)\}$.

3. OPTIMUM TRANSFORM ORDER

The general expression of a linear chirp signal x(t) can be expressed as:

$$x(t) = \exp[j^2\pi(at^2 + bt + c)] \tag{9}$$

Where parameter , *a* is the chirp rate in Hz/second, *b* is the initial frequency in H_z, *c* is the initial phase in degree, and *t* is the time in seconds.

The phase $\phi(t)$ of the signal is defined as:

$$\phi(t) = at^2 + bt + c \tag{10}$$

The instantaneous frequency $\phi'(t)$ is the derivative of the phase:

$$\phi'(t) = 2at + b \tag{11}$$

The chirp-rate of the signal is the derivative of the instantaneous frequency $\phi'(t)$ and is equal to 2a. The optimum transform order α_{opt} is that value of α for which the FrFT produced the most compact form (maximum compression) of the given chirp signal in the fractional domain. The optimum transform order is related to the chirp rate of the signal. In discrete form, the optimum transform order can be defined as [14]:

$$\alpha_{opt} = 2 - \frac{2}{\pi}\tan^{-1}\left(\frac{\frac{f_s^2}{N}}{2a}\right) \tag{12}$$

Where ϕ_{opt} is optimum transform order, f_s is sampling frequency, N is number of samples, and a is chirp rate.

There are two main applications of FrFT in ultrasound imaging. In first application the FrFT can be used as an alternative to a matched filter for chirp pulse-compression [17]. Whereas in second application the FrFT can be used as a filtering tool to extract the overlapped chirp signals [18-19].

3.1 Proposed Method

The block diagram of the proposed method is shown in Fig. 1. In the proposed method, the extraction of the overlapped second harmonic component is performed using the FrFT. In the first step, the nonlinear received signal is transformed into the fractional domain existing between time and frequency domain sat the optimum transform order α_{opt}. The second step is to perform filtration in the fractional domain in order to extract the desired second harmonic and completely suppress the fundamental frequency component. In this paper, a rectangular window is used for the extraction of second harmonic component in the fractional domain. The rectangular window is selected so that it keeps all the information of nonlinear echoes present in the second harmonic without adding significant distortion in the time, frequency or composite fractional domain [18]. After the extraction of second harmonic component, the extracted harmonic chirp signal is transformed back to the time domain in order to perform pulse compression using the HMF (Harmonic Matched Filter).

3.2 Simulation

In order to get nonlinear harmonic data, simulations were performed using a commercially available FEA (Finite Element Analysis) package PZFlex (Weidlinger Associates Inc., CA, USA). In all simulations, a 2D PZFlex model was constructed. The model dimensions were 20mm by 50mm, damping frequency of 2MHz and the number of elements per wavelength was 20. Also water was selected as a propagation medium. For excitation, no transducer model was built but only the pressure waves having a peak pressure of 1MPa were directly applied in the model. The data obtained from the simulation were further processed in MATLAB (MathWorks Inc., Natick, MA, USA) for FrFT filteration and pulse compression.

4. SIMULATION RESULTS AND DISCUSSION

In the simulation, three linear chirps having duration of 10μsec, centre frequency of 2.25 MHz, with -6dB fractional bandwidths of 22.6, 44.2 and 66.7% respectively were used as excitation signals. In the design process of the excitation signals, the Hann window was also applied in order to reduce ripples in the amplitude spectrum. The 22.6% fractional bandwidth chirp excitation signal and its magnitude spectrum are shown in Fig. 2.

The nonlinear received signal and associated frequency spectrum is shown in Fig. 3. In addition to the fundamental component, the frequency spectrum of the received signal clearly shows the existence of second and third harmonic components, which were produced due to the nonlinear propagation of ultrasound waves into the water at high acoutic pressure [20].

The filtering and pulse compression process of nonlinear received data were carried out using MATLAB (MathWorks Inc., Natick, MA, USA). In order to extract the second harmonic component from the signal shown in Fig. 3, the signal is transformed into the fractional Fourier domain at optimum value of alpha. The FrFT of the nonlinear received signal is shown in Fig. 4. The FrFT is computed at alpha = 1.02. This optimum value of alpha is computed using Equation (12), where the parameters a = 50*109 Hz/sec, N = 8200 and fs = 157.26 MHz. In this paper, the FrFT is computed using the algorithm produced by Ozaktas, et. al. [21].

The Fig. 4 clearly shows the existence of three signal harmonic components in the FrFT domain. To extract the second harmonic component, filtering was performed by

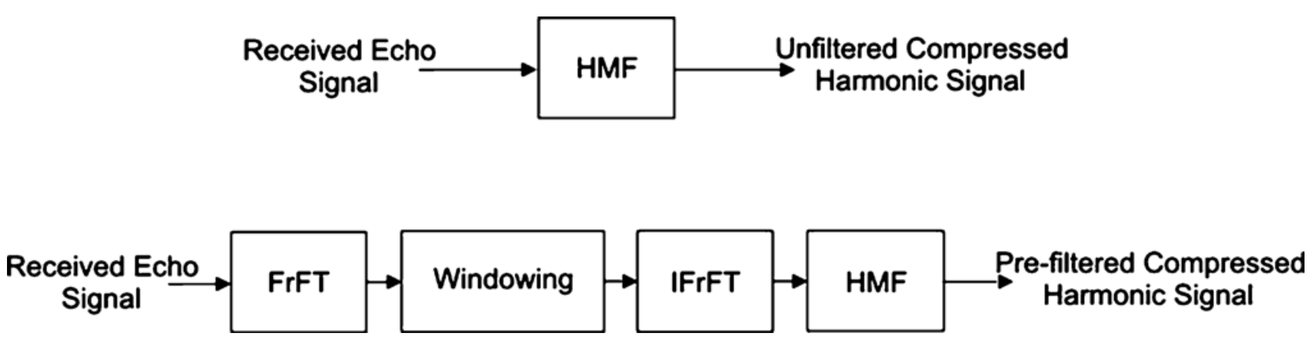

FIG. 1. BLOCK DIAGRAM OF THE RECEIVING SYSTEM SHOWS THE PROCESSING OF ECHO SIGNAL IN CONVENTIONAL (TOP) AND PROPOSED (BOTTOM) METHODS

applying the rectangular window around the corresponding peak in the FrFT domain (second peak in Fig. 4). After the extraction of second harmonic component, the time domain signal was obtained by taking their inverse FrFT at minus optimum alpha = -1.02 [15]. The extracted second harmonic chirp signal and associated amplitude spectrum are shown in Fig. 5.

Similarly simulations were also performed to get the nonlinear harmonic data using 44.2 and 66.7% fractional bandwidth chirp excitations. The amplitude spectra of the nonlinear received signals under 44.2 and 66.7% fractional bandwidth chirp excitations are shown in Fig. 6. It is clear in Fig. 6 that the first and third harmonics are overlapped with the second harmonic component. It is also observed that the overlapping amongst the nonlinear harmonic components is significantly increased by increasing the fractional bandwidth of the chirp excitation signal.

The FrFT was used to extract the overlapping second harmonic component from the nonlinear signals which are received under 44.2 and 66.7% fractional bandwidth chirp excitations.The optimum values of the transform order for these signals are computed at alpha = 1.04 and 1.06 respectively. After the extraction of second harmonic components in the FrFT domain, the extracted signals are transformed back to the time domain. The extracted second harmonic components in the frequency domain are shown in Fig. 7.

The effect of FrFT has been evaluated by performing the pulse compression of the extracted second harmonic chirp signal and to recover the axial resolution. The comparison is also made by performing the pulse compression of unfiltered received signal. The pulse compression process is performed using the HMF. The HMF was designed with twice the centre frequency and fractional bandwidth of the excitation signal [10]. Fig. 8 shows the comparison between FrFT pre-filtered and unfiltered compressed harmonic chirp signals.

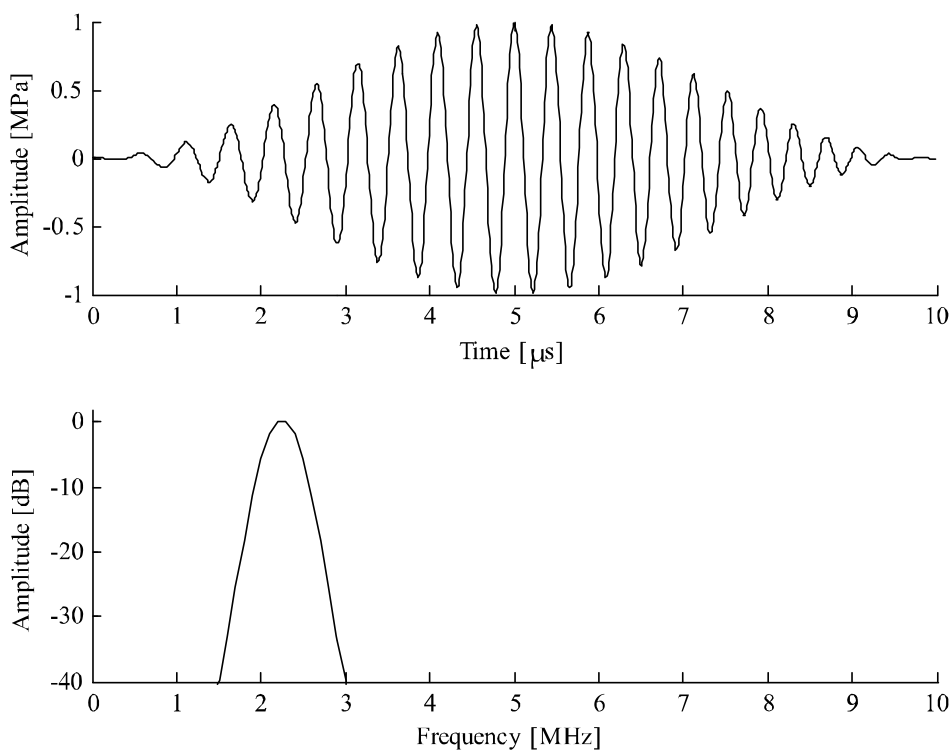

FIG. 2. CHIRP EXCITATION SIGNAL WITH A FRACTIONAL BANDWIDTH OF 22.6% (TOP) AND ASSOCIATED AMPLITUDE SPECTRUM (BOTTOM)

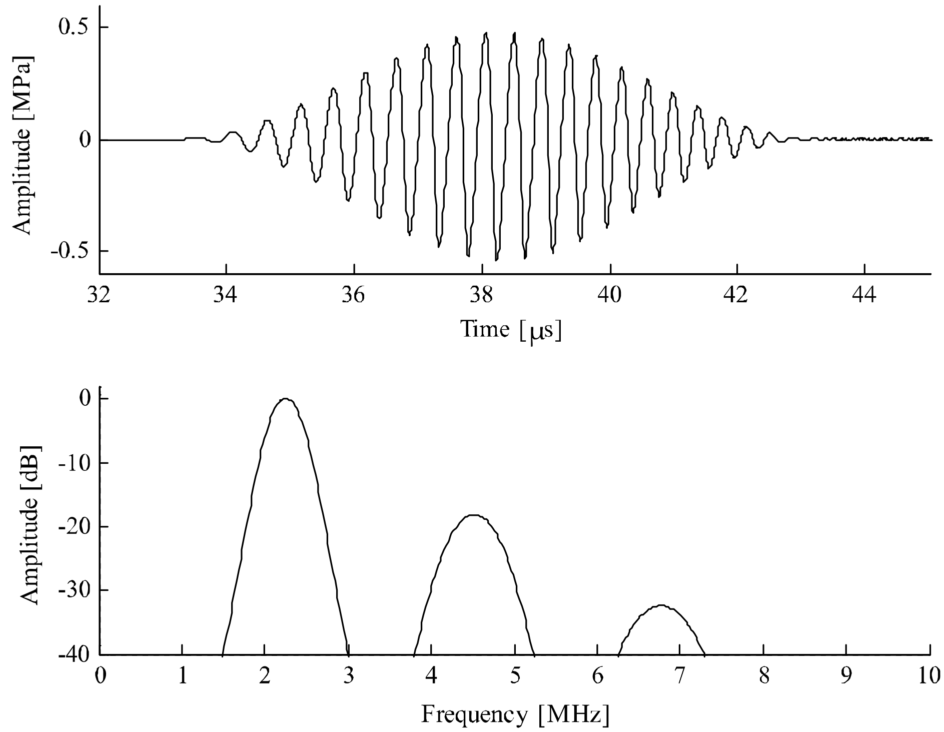

FIG. 3. NONLINEAR RECEIVED SIGNAL (TOP) AND ASSOCIATED AMPLITUDE SPECTRUM (BOTTOM)

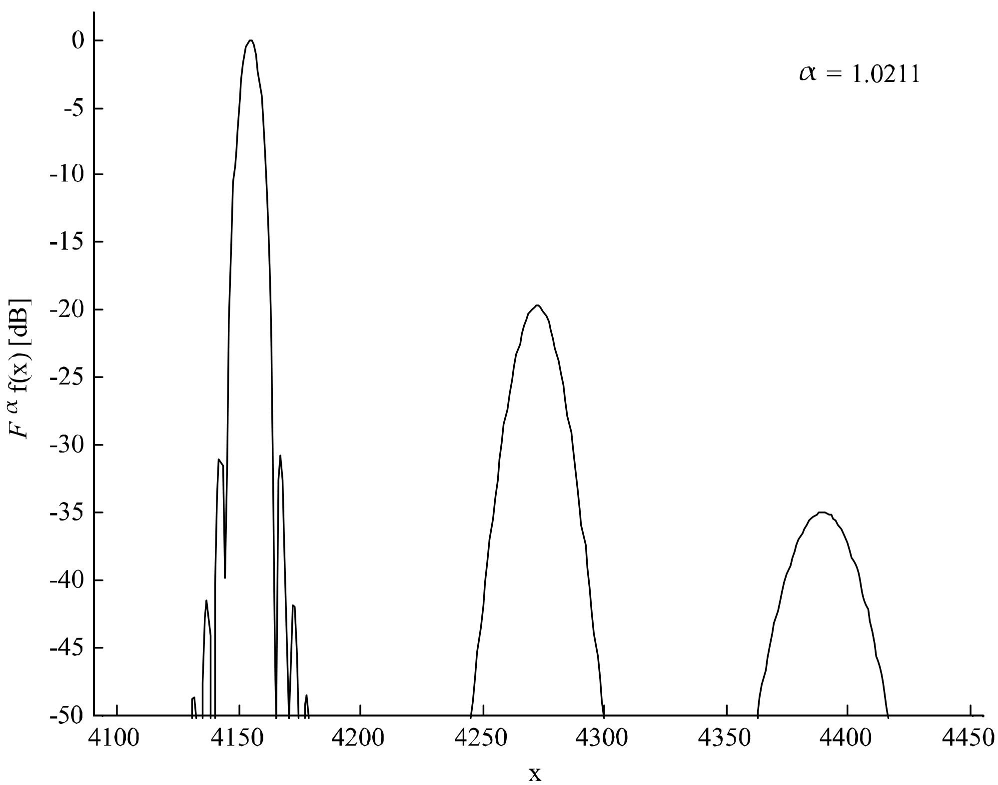

FIG. 4. THE FRACTIONAL FOURIER TRANSFORMED SIGNAL COMPUTED AT α=1.0211

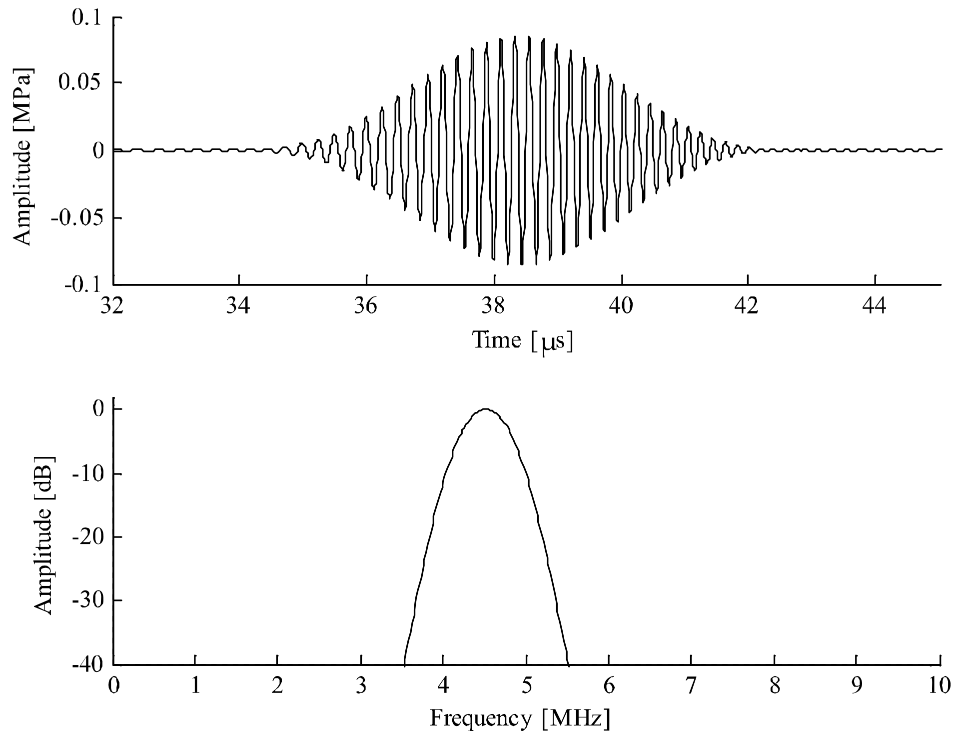

FIG. 5. THE EXTRACTED SECOND HARMONIC CHIRP SIGNAL (TOP) AND ASSOCIATED AMPLITUDE SPECTRUM (BOTTOM)

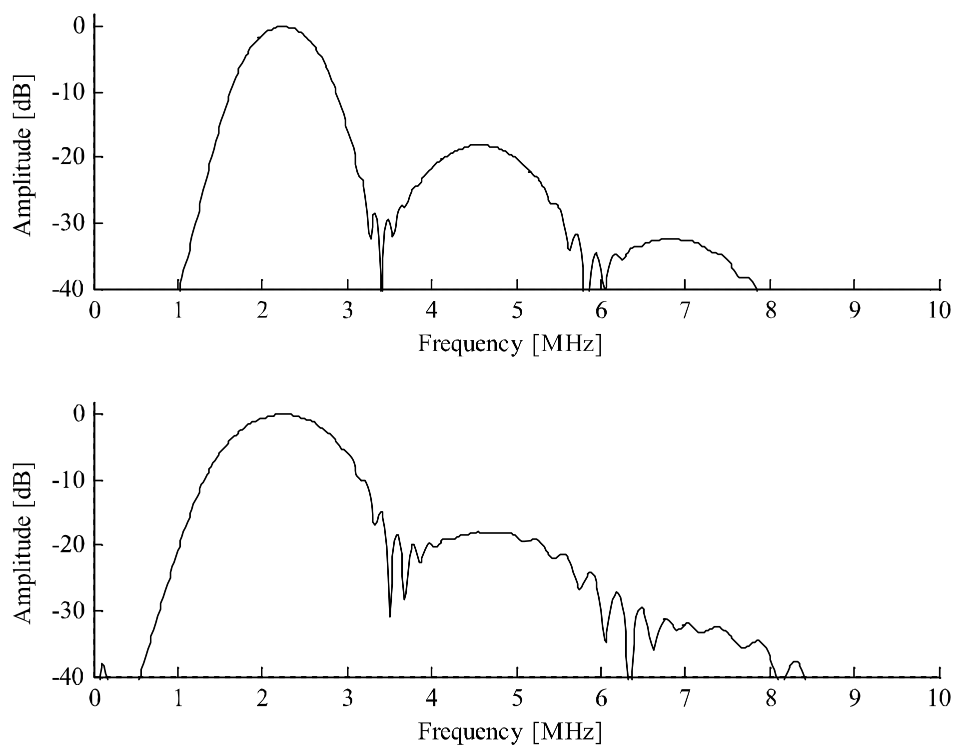

FIG. 6. THE AMPLITUDE SPECTRA OF THE NONLINEAR RECEIVED SIGNALS UNDER CHIRP EXCITATION WITH -6dB FRACTIONAL BANDWIDTH OF 44.2% (TOP) AND 66.7% (BOTTOM) RESPECTIVELY

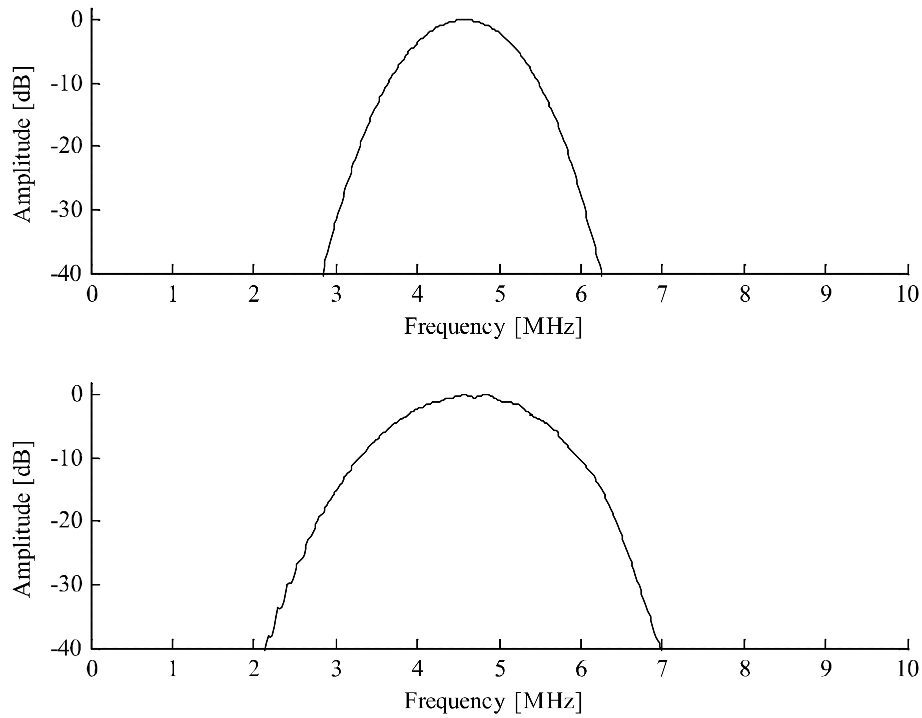

FIG. 7. THE AMPLITUDE SPECTRA OF EXTRACTED SECOND HARMONIC CHIRP SIGNALS OBTAINED UNDER CHIRP EXCITATION WITH -6dB FRACTIONAL BANDWIDTH OF 44.2% (TOP) AND 66.7% (BOTTOM) RESPECTIVELY

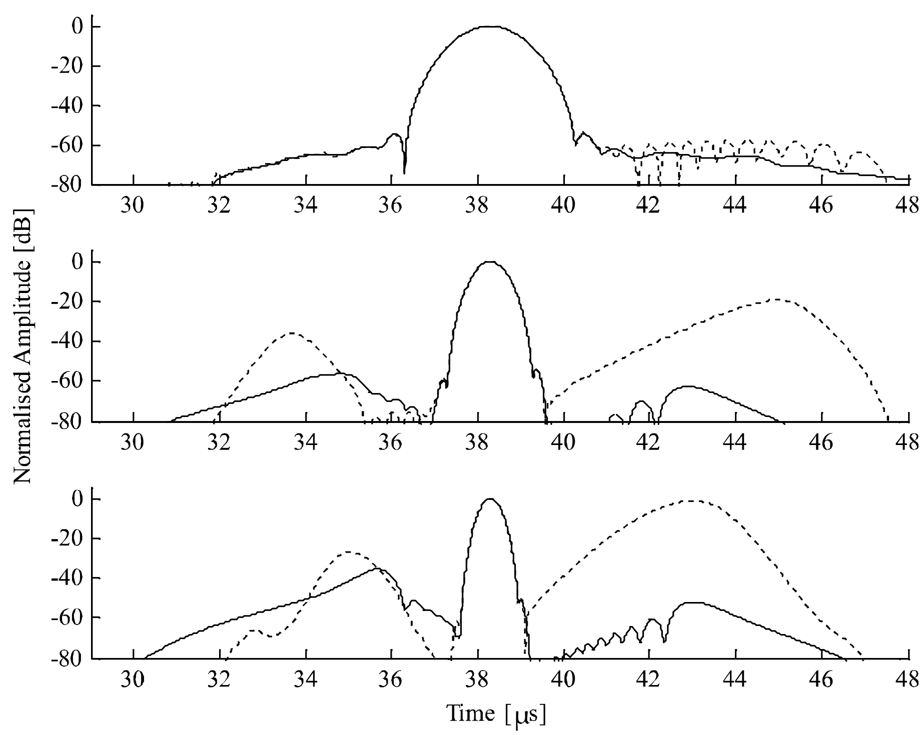

FIG. 8. THE COMPARISON OF UNFILTERED (DASHED LINE) AND FRFT PRE-FILTERED (SOLID LINE) HARMONIC MATCHED FILTERED COMPRESSED SIGNALS OBTAINED USING CHIRP EXCITATION WITH -6dB FRACTIONAL BANDWIDTH OF 22.6% (TOP), 44.2% (MIDDLE) AND 66.7% (BOTTOM) RESPECTIVELY

The MLW (Main Lobes Width) of the compressed harmonic chirp signals are shown in Table 1. It is found that MLW of the unfiltered and FrFT pre-filtered compressed signals are same. Therefore, FrFT has no effect on the MLW of the compressed signal and hence on the axial resolution. The RSLL of the compressed harmonic chirp signals are shown in Table 2. It is observed that up to 35.5dB reduction in the RSLL has been found in the FrFT pre-filtered compressed chirp signal when compared with unfiltered compressed signal.The RSLL are significantly reduced in the FrFT pre-filtered compressed signal which potentially improves the signal detection and contrast resolution.

5. CONCLUSION

Penetration depth and axial resolution are always desired in ultrasound harmonic imaging. Linear frequency modulated chirp signals have a potential to achieve these goals whilst it requires only single transmission which eliminates the motion artifacts and may enhance the frame-rate of the system. We have presented a technique which uses broadband chirp signals as excitation with the FrFT on the receiving side for the extraction of second harmonic component under overlapped condition. Comparisons were also made by performing the pulse compression of the FrFT pre-filtered second harmonic chirp signal and the unfiltered received signal using harmonic matched filter. Our results indicate up to -35.5dB reduction in the RSLL has been found in the FrFT pre-filtered compressed harmonic chirp signal compared to the unfiltered compressed signal. Also the MLW of the FrFT filtered compressed chirp signal was similar to the mainlobe of the unfiltered compressed chirp signal. The reduction of RSLL in the compressed harmonic chirp signal using the FrFT filtering can significantly improve the image resolution.

TABLE 1. THE MLW OF COMPRESSED HARMONIC CHIRP SIGNALS

Fractional Bandwidth (%)	Mainlobe Width (μsec)	
	Unfiltered Compressed Signal	FrFT Pre-Filtered Compressed Signal
22.6	2.68	2.68
44.2	1.36	1.36
66.7	0.91	0.91

TABLE 2. THE RANGE SIDELOBES LEVEL OF COMPRESSED HARMONIC CHIRP SIGNALS

Fractional Bandwidth (%)	Rang Sidelobes Level (dB)	
	Unfiltered Compressed Signal	FrFT Pre-Filtered Compressed Signal
22.6	-53.8	-54.4
44.2	-19.5	-56.6
66.7	-1.1	-36.6

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