SNR and BER Models and the Simulation for BER Performance of Selected Spectral Amplitude Codes for OCDMA

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ABSTRACT

Many encoding schemes are used in OCDMA (Optical Code Division Multiple Access Network) but SAC (Spectral Amplitude Codes) is widely used. It is considered an effective arrangement to eliminate dominant noise called MAI (Multi Access Interference). Various codes are studied for evaluation with respect to their performance against three noises namely shot noise, thermal noise and PIIN (Phase Induced Intensity Noise). Various Mathematical models for SNR (Signal to Noise Ratios) and BER (Bit Error Rates) are discussed where the SNRs are calculated and BERs are computed using Gaussian distribution assumption. After analyzing the results mathematically, it is concluded that ZCC (Zero Cross Correlation Code) performs better than the other selected SAC codes and can serve larger number of active users than the other codes do. At various receiver power levels, analysis points out that RDC (Random Diagonal Code) also performs better than the other codes. For the power interval between -10 and -20 dBm performance of RDC is better ZCC. Their lowest BER values suggest that these codes should be part of an efficient and cost effective OCDM access network in the future.

Key Words: **Optical Code Division Multiple Access Network, Long Reach Passive Optical Network, 2-D Coding, Optical Orthogonal Codes.**

1. INTRODUCTION

OCDMA is attracting more and more interest day-by-day due to its inherent characteristics [1-3]. It is a strong contender for future access technologies in PON (Passive Optical Network). It is also a favorable candidate for providing broadband related telecom solutions in the last mile section [4]. However, its acceptance is restricted due to MAI [5-6]. This type of interference causes degradation in the signal power received by a user. MAI results when signals from neighboring users are simultaneously transmitted and cause interference in the information signal for intended subscriber. This added noise signal degrades the reception of the information signal and increases the bit error rate. The impact is more pronounced when there is large number of active subscribers transmitting simultaneously.

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To reduce this effect, OCDMA codes having orthogonality are used in the network. This orthogonality results in elimination of this nuisance. However, OCDMA codes have their own limitations. Therefore, choosing the code with good performance is not a simple task. The codes are characterized by many features such as the cardinality of the code, i.e. how many simultaneous or active subscribers can be supported with this code family, length of code, code weight (number of marks in the code), the amount of frequency bins having a particular spectral width and the last but not the least, the in-phase cross correlation. This parameter gives the measure of orthogonality of the codes within the same family. It can be defined as number of wavelengths common to two codes.

Spectral amplitude coding is one of the prevalent schemes used to encode and decode codes for OCDMA networks. It was introduced in 1990s [7]. It is the part of future passive optical access networks because it is cost efficient as well as spectral efficient [8].

SAC is beneficial in reducing MAI. For a code of given length, code weight and the number of subscribers, cross-correlation is the only leftover defining factor of MAI. Therefore, code family with lowest cross-correlation values is preferred in the network, where this interference is intended to be completely eliminated. SAC scheme uses differential or balanced detection to perform the above task. In this type of detection, normal decoding and referential decoding are used to cancel out the intruding noise from other users.

Various code families have been used. For example quasi-orthogonal code families like m-sequence and Walsh-Hadamard were used in the experimental demonstration described in [9]. Later on, MQC (Modified Auadrature Congruence) codes were tested in [10]. Zhou, et. al. [11] used BIBD (Balanced Incomplete Block Design) code to describe their SAC scheme. In 2002, they used another code called MFH (Modified Frequency Hopping) code [3]. A SAC code called modified double weight code was developed in 2004 [12]. These codes can easily be implemented. For this purpose optical filters can be applied to obtain desired spectral components from the spectrum by slicing the required spectral width for the code.

In this paper, some selected SAC codes are evaluated for their performance against the interference and their robustness against the noise. In the first part, codes and their constructions are briefly described. In the later part of this paper, the analytical appraisal of their utility is carried out and presented in the pictorial form. In order to calculate SNR, three major ratios are considered namely PIIN, shot noise and thermal noise. Other noises are neglected to simplify the calculations. Therefore, noise figure consists of:

$$\left\langle N_{Noise}^{2} \right\rangle = \left\langle i_{PIIN}^{2} \right\rangle + \left\langle i_{Shot}^{2} \right\rangle + \left\langle i_{Th}^{2} \right\rangle \quad (1)$$

2. CODE CONSTRUCTION

In this section, construction methods, properties of codes and their formulae for SNR are briefly described to give a better understanding. The SAC will be applied to an OCDMA network. A general view of the SAC OCDMA system setup is given in Fig. 1.

2.1 Hadamard Code

This was originally a bipolar code consisting of (+1, -1) and was used in wireless communication for FEC (Forward Error Correction). In CDMA it was developed from Walsh matrices. Codes are derived from the rows of these matrices. A general matrix is given as:

$$H(i+1) = \left[\begin{array}{c|c} H(i) & H(i) \\ \hline H(i) & -H(i) \end{array}\right] \tag{2}$$

where i=0,1,2,…. Each row in the matrix forms a code word.

In OCDMA, it is used in unipolar format, i.e. 1 and 0 as the code alphabet instead of +1 and -1. This type of code is not fully orthogonal and cross-correlation between two rows is not zero. Therefore this kind of code is termed as quasi-orthogonal and the value of cross correlation depends upon the length of message m (2^{m-2}).

SNR is calculated with the help of the limit defined in [10].

$$SNR_H = \frac{\Delta v}{K(K+1)B} \tag{3}$$

Where Δv is the spectral width as described in Table 1, K is the number of simultaneous users and B is the electrical bandwidth.

2.2 BIBD Code

BIBD code is developed using block design based on combinatorial construction theory. In [13], Hall defines it in these words: *"A balanced incomplete design is an arrangement of v distinct objects into b blocks such that each block contains exactly k distinct objects; each object occurs in exactly r different blocks, and every pair of distinct objects a_i, a_j occurs together in λ blocks."*

Codes are constructed with the help of this theorem and projective geometry PG(m,q) is used where m indicates the number of dimensional vectors and *q* is prime power of p and $q=p^n$. p is the prime number in the Galois field. Using this design, we get this code $\{(q^{m+1}-1)/(q-1), (q^{m-1})/(q-1), (q^{m-1}-1)/(q-1)\}$ [11].

SNR for this code is defined by this relation:

$$SNR_{BIBD} = \frac{2q\Delta v}{BK\left(\frac{K}{2}+q-1\right)} \tag{4}$$

2.3 Modified Quadrature Congruence Code

MQC family is based on (p^2+p, p+1, 1) where p is a prime number and an element of Galois field. Its construction details are given in [14].

Properties of MQC and other codes are summarized in Table 1. Its SNR is defined as:

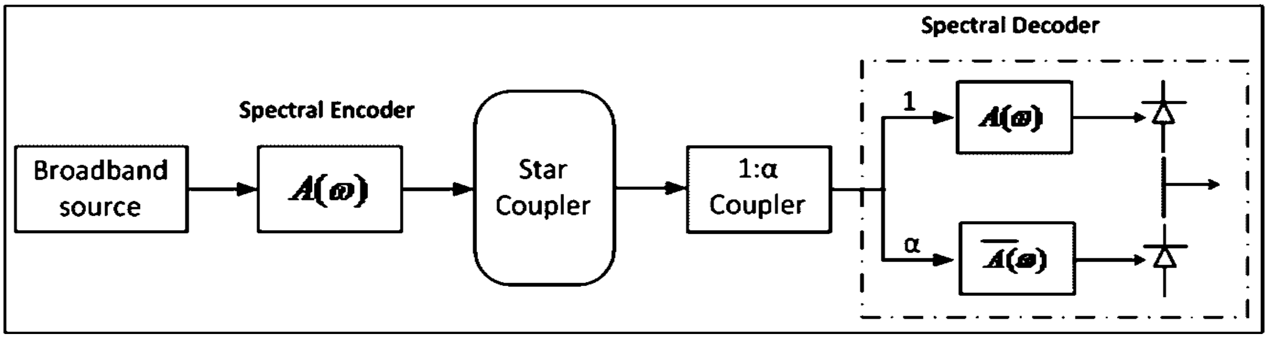

FIG. 1. SPECTRAL AMPLITUDE CODING

$$SNR_{MQC} = \frac{\frac{\Re^2 p^2}{p^2}}{B\Re^2 p_{sr}^2 K\left[\frac{\frac{(K-1)}{p}+p+K}{2\Delta v(p+1)p^2}\right] + eB\Re P_{sr}\frac{(p-1+2K)}{(p^2+p)} + \frac{4K_b T_n B}{R_L}} \quad (5)$$

Here $\Re$ stands for the responsivity of the photdetector. It is the ratio of the generated photocurrent and incident optical power. K is the number of active users and p is the prime number used to generate codes. The rest of symbols in the equation are defined in Table 2.

2.4 Multi-Frequency Hopping Code

This code was proposed by Wei, et. al. [15]. This is a modified version of code presented in [8]. This is developed by padding technique. It means that some zeros and ones are added to the original codes of BIBD family and newer code type called multi frequency hopping code is produced. By padding, code length is increased. Further construction details can be found in [3].

SNR for this code used in the analysis is:

$$SNR_{MFH} = \frac{2Q\Delta v}{BK\left(\frac{K}{2}+Q-1\right)} \quad (6)$$

where Q is the prime number.

2.5 Modified Double Weight Code

Earlier coding schemes stated above have some problems regarding their complicated construction methods. Besides code lengths are large when subscribers increase. Codes from easy constructional point of view are required that are also good and effective in cancelling out MUI besides possessing shorter code lengths. It means their cross correlation values should be the lowest for their acceptability and adaptability in spectral amplitude coding schemes. Aljunid, et. al. [12] proposed a new code family called double weight of code weight equivalent to 2 and

TABLE 1. PROPERTIES OF SELECTED SPECTRAL AMPLITUDE CODES

No.	Code	Length (N)	Code weight (w)	Cross-correlation (λ)	Cardinality
1.	Hadamard	2^m	2^{m-1}	2^{m-2}	2m-1
2.	BIBD	$(q^{m+1}-1)/(q-1)$	$(q^{m-1})/(q-1)$	$(q^{m-1}-1)/(q-1)$	q^2+q+1
3.	MQC	p^2+p	p+1	1	p^2
4.	MFH	Q^2+Q	Q+1	1	Q^2
5.	MDW	$N=3n+8/3[\sin(K\pi/3)]^2$	4	1	N
6.	ZCC	4N	4	0	N
7.	RD	N=K+2W-3	W	1	K
8.	DCS	$N = \sum_{i=1}^{W-1}(2^i + D)$	W	1	N

MDW (Modified Double Weight) code that has code weight greater than 2.

DW is simply a KxN matrix, where N stands for the code length and K (number of rows) indicates the number of users. Both are related as:

$$N = 3K + \frac{8}{3}\left[\sin\left(\frac{K\pi}{3}\right)\right]^2 \tag{7}$$

The code construction details are given in [11]. Its SNR is defined in Equation (8):

$$SNR_{MDW} = \frac{2\left(\frac{W}{\lambda} - 1\right)\Delta v}{BK\left(\frac{K}{2} + \frac{W}{\lambda} - 2\right)} \tag{8}$$

Here W stands for weight of the OCDMA code and K is the number of live users. The rest of symbols are given in Table 2.

2.6 Zero Cross-Correlation Code

The major source of noise at the receiver side is PIIN. When multiple transmitters are sending incoherent signals and the addition of these signals reach at a particular photodiode, it results in intensity noise [16]. This noise is directly influenced by MAI because of common spectral components among the users [14]. To address this issue, ZCC code was designed by Anuar, et. al. [17]. It can eliminate MAI and thus can suppress this noise. By this, it improves the efficiency of spectral amplitude coding system by lowering the BER by reducing PIIN and increasing the users simultaneously.

SNR used in this analytical survey is given as:

$$SNR_{ZCC} = \frac{\dfrac{2\Re^2 p_{sr}^2 W^2}{L^2}}{\dfrac{p_{sr} e B \Re^2 \left(K - 1 + W\right)}{L} + \dfrac{4K_b T_n B}{R_L}} \tag{9}$$

TABLE 2. SYMBOLS AND VALUES OF PARAMETERS USED IN SAC OCDMA

Symbol	Parameter	Value
η	photodetector quantum efficiency	0.6
Δv	linewidth of broadband source	3.75THz
λ_0	operating wavelength	1550nm
B	electrical bandwidth	80~311MHz
T_n	receiver noise temperature	300K
R_L	receiver load resistor	1030Ω
e	electron charge	$1.6\text{x}10^{-19}$C
h	Plank's constant	$6.66\text{x}10^{-34}$Js
K_b	Boltzmann's constant	$1.38\text{x}10^{-23}$J/K
P_{sr}	Power of the light source at the receiver	-10dBm

Here, $\Re$ stands for the responsivity of the photdetector. It is the ratio of the generated photocurrent and incident optical power. It can be defined by Equation (10).

$$\Re = \eta \frac{e}{hv} \tag{10}$$

where η is the quantum efficiency, e is the electron charge, and *hv* is the photon energy. W in Equation (9) is the code weight, K is the number of active users and N is the code length.. The remaining symbols are described in Table 2.

2.7 Random Diagonal Code

There are two ways by which MAI can be reduced. One is to design codes with very low cross correlation and the other option is to develop a better detection technique. This is designed to eliminate MAI because it has ZCC and it consists of two parts: data segment and code segment [18]. Construction details are available in the referred article. For this code Equation (11) gives the SNR expression.

$$SNR_{RDC} = \frac{\left(\frac{2\Re P_{sr} W}{N}\right)^2}{\frac{2eBWP_{sr}\Re}{N} + \frac{B\Re^2 WK(K-1+W)}{2N^2\Delta v} + \frac{2K_b T_n B}{R_L}} \tag{11}$$

where N is the code length. For the rest of the symbols refer Table 2.

2.8 Dynamic Cyclic Shift Code

DCS code has one advantage over other spectral amplitude codes. Its code length matches with the number of subscribers [19]. Also its cross-correlation is less than 1(λ<1). In this coding scheme, weight needs not to be enhanced if the number of users is increased. For doing so, dynamic part is enlarged. The details of the construction of this code can be viewed in the above reference.

SNR used in this regard is given in Equation (12).

$$SNR_{DCS} = \frac{\left(\frac{2\Re P_{sr} W}{N}\right)^2}{\frac{eB\Re P_{sr}(W+3)}{N} + \frac{B\Re^2 K(W+W)}{2N^2\Delta v} + \frac{4K_b T_n B}{R_L}} \tag{12}$$

3. MEASUREMENT OF BER

BER in an OCDMA system depends largely on the user data rate (R_b), the number of active or simultaneous users in the system and orthogonality between any two codes of the code family used in the system [20]. Orthogonality of codes translates in the pdf (probability density function). To calculate BER, a better estimate of probability distributions of the noise or interfering signals is required. This pdf depends upon length and weight of code. Probability density function is a statistical measurement of the distribution of probabilities of a random variable.

If BER of one subscriber out of N users in the system is to be calculated, probability density function of the remaining users (N-1) is to be determined. However this leads to a rigorous and complicated calculation if N gets larger. Therefore only upper or lower bounds of this measure are estimated in general and applied for evaluation. Here in this analysis same assumptions are made as given in [14]. One more assumption made here is that all noises have Gaussian distribution.

The photocurrent intensity of the receiving photodiode is affected by the multiple noises, like intensity induced noise, shot noise, thermal noise and dark current. If dark current is neglected, variance in photocurrent intensity can be defined as:

$$\langle i^2 \rangle = 2eIB + I^2 B\tau_c + \frac{4K_b T_n B}{R_L} \tag{13}$$

where *i* stands for the average photocurrent received by the photodiode. The rest of symbols used in Equation (13) are illustrated in Table 2.

In Equation (13), the first part represents formulation of shot noise, the second one relates phase induced intensity noise (PIIN), and the last component describes the thermal noise.

For this analysis, various parameters are used as given in Table 2. First SNR was calculated. SNR can be related as in the following equation:

$$SNR = \frac{(I_d)^2}{\langle i^2 \rangle} \tag{14}$$

Here I_d is the current detected minus noises at the photodiode and denominator part constitutes the noise part. For calculating BER, Gaussian distribution is assumed and following relation is used:

$$BER = \frac{1}{2} erfc\sqrt{\frac{SNR}{8}} \tag{15}$$

Where *erfc* is an error function.

4. RESULTS AND PERFORMANCE EVALUATION

All the above stated codes were used in the analysis and their BER against the number of simultaneous users were calculated. Fig. 2 gives the performance curves of the codes. As Fig. 1, ZCC performs much better than the other codes because it has ZCC and the resultant PIIN component is neglected in the determination of BER for this code. Performance of ZCC codes is superior because these do not have spectral components overlapping with each other. Therefore, due to good structural properties of this code, it rates high above among the rest. The performance of Hadamard code lags behind as

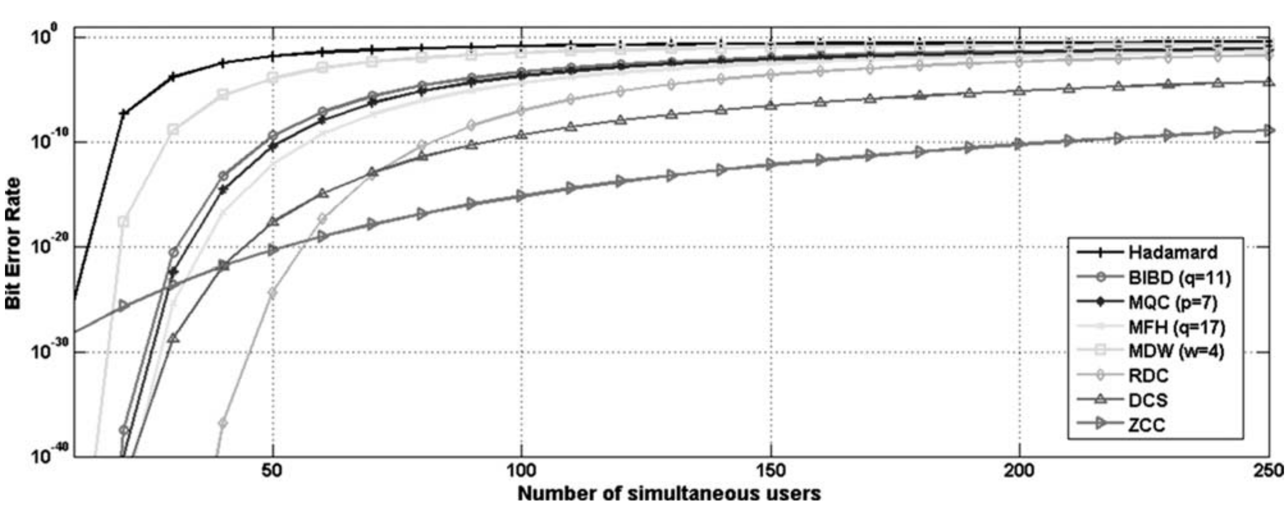

FIG. 2. BER VERSUS SIMULTANEOUS USERS OF SAC CODES

theoretically expected because it has high cross-correlations value. The construction of MDW is easy but its performance is not better than MQC and MFH. RDC is placed behind ZCC indicating better performance against other codes except ZCC.

If received power is used for checking the performance of the codes, while keeping the number of active users constant, result can be obtained in the form of performance curves shown in Fig. 3. Here four codes MQC, RDC, DCS and ZCC are used for analysis. The number of simultaneous users is fixed as 100 and performance is determined on various levels of receiver power. From Fig. 3, it can be established that at higher levels of power, i.e $P_{sr} > -25$ dBm, codes have different performing characteristics. But below this level, all codes converge to same performance level. For the power interval of -10 and -15dBm, again ZCC display a better performance as compared to the rest of three codes. However between interval of -15 and -20 dBm, RDC emerges as a code with better lowest BER values. Since performance of ZCC is limited by two noise components: shot noise and thermal noise. Thermal noise is constant and is not affected by various power levels. But shot noise depends upon the average photocurrent current received by the photodiode receiver. The reduced photocurrent results in reduced shot noise at the receiver. Therefore, for this interval, performance of RDC is much higher than other codes included in the analysis.

From Figs. 2-3 displaying performance curves of selected SAC codes, it can be concluded that ZCC and RDC are better performing codes. These codes can be utilized in the OCDMA access networks to improve the efficiency. Also these can be made part of the future passive optical access network because of their spectral efficiency and robustness against noises. These codes will enhance the cost effectiveness as well as improve the efficiency of the network.

5. CONCLUSION

Various spectral amplitude codes are studied for evaluation with respect to their performance against three noises namely shot noise, thermal noise and PIIN. Their SNRs are calculated and BERs are computed using Gaussian distribution assumption. After analyzing the results mathematically, it is concluded that ZCC performs better than the other selected SAC codes and can serve larger number of active users than the other codes do. At various receiver power levels, analysis points out that RDC also performs better than the other codes. For the power interval

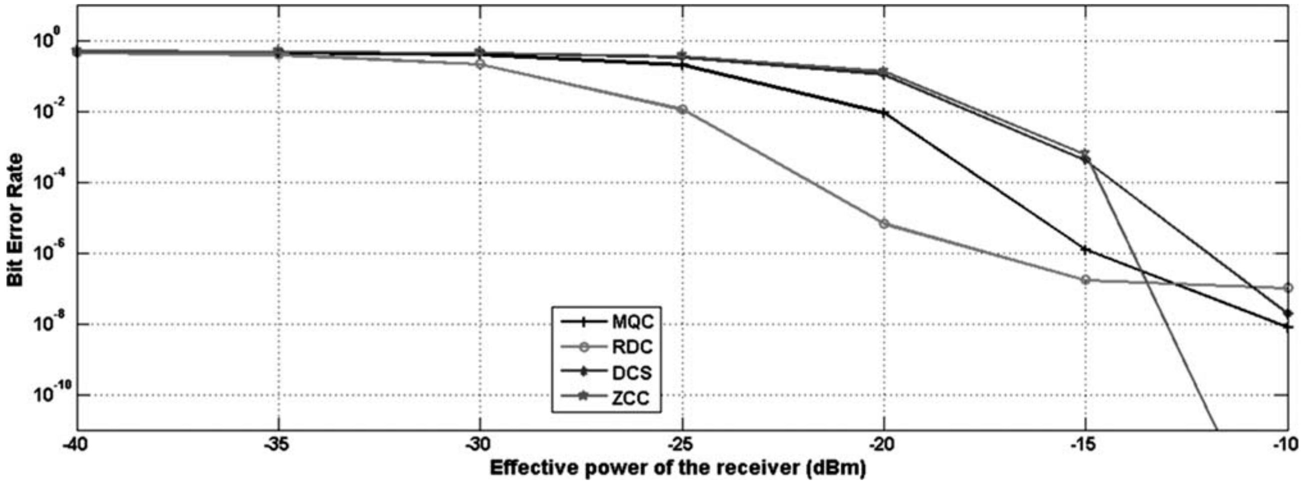

FIG. 3. BER VERSUS RECEIVED OPTICAL POWER

between -10 and -20 dBm performance of RDC is ahead of that of ZCC. Their lowest BER values suggest that these codes should be part of an efficient and cost effective OCDM access network in the future.

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