Stiff Columns as Liquefaction Mitigation Measure for Retrofit of Existing Buildings

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ABSTRACT

In this paper, ground reinforcement with jet grouted columns under shallow foundations of existing buildings was analysed using numerical modelling. This study is related with ground reinforcement by installing stiff jet grouted columns around the shallow foundations of existing building when the foundation soil is liquefied during an earthquake. The isolated shallow square footing pad supporting a typical simple frame structure was constructed on the reinforced ground with stiff jet grouted column rows at the shallow depth from the ground surface. This soil-structure system was modelled and analyzed as plane-strain using the FLAC (Fast Lagrangian Analysis of Continua) 2D dynamic modelling and analysis software. The results showed that liquefaction-induced large settlement of shallow foundation of existing building can be reduced to tolerable limits by applying ground reinforcement with continuous rows vertical jet grouted columns adjacent to footing pad.

Key Words: **Liquefaction, Jet Grouted Columns, Retrofit, Adjacent Columns, Existing Buildings, Mitigation, Numerical Modelling.**

1. INTRODUCTION

Karachi a southern city of Pakistan is lying in the region most vulnerable to earthquakes which usually originate from epicentre at Gujrat fault. In coastal areas of Karachi like DHA (Defence Housing Authority) and Clifton area soil deposits may be susceptible to liquefaction in the event of earthquake. The existing buildings in this area may likely be damaged due to liquefaction-related large settlements.

The existing buildings with shallow foundations at the site where liquefiable sand deposits are present and the ground water table is at or near the foundation level, are vulnerable to damage because of liquefaction-related large permanent settlements. These liquefaction-related permanent large settlements are initiated at the ultimate bearing capacity (shear failure). This type of shear failure occurs due to loses stiffness of soil as a result of liquefaction. Besides, when the soil is reconsolidated or densified as result of pore water pressure dissipation, volume change of the soil, incremental settlements during and after the cyclic loading occur. In earthquakes like Niigata, Japan, [1], Dagupan City; Chi-Chi, Taiwan, [2-3] and Kocaeli, Turkey, [4] such type of response of existing buildings was observed. In addition, there are many important historic buildings in Greece and Italy which may be damaged due to potential liquefaction in future earthquakes. Currently, there is no proven design for retrofit of existing buildings due to liquefaction [5].

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Grouting is a technique which is used for retrofit of existing shallow foundations because it can be performed without excessive noise and vibration [5]. Micropiles have been applied for many applications which include seismic retrofit. They may be suitable for existing structures because they can be applied with low level of vibration and noise. Further, micropiles can be installed at different angles and also in areas where access is restricted and headroom is very low [5].

In past research on the solidification principle of ground improvement has been focused on the cemented zones beneath and around the footing pad of structure. In this way, treating most of the area of the structure, which may not be cost-effective and easy. Besides, treating all the area beneath the structure may amplify the motions towards the structures [6-7]. Also providing grouting beneath the footing may not be feasible for existing structure.

The ground reinforcement with stiffer high modulus jet grouted/deep mixing column rows, requires relatively small replacement area in the ground. This treatment has recently demonstrated its performance in earthquakes such as Kocaeli, Turkey, and Kobe, Japan, [6-7] may be more cost effective and easier to install. Further, with relatively less replacement area, motions transmitted towards structures may be relatively small.

Further, little work has been focussed on the optimum geometry of deep mixing/jet grouted columns in the ground for limiting the settlements of shallow foundations to meet the tolerable limits of the existing buildings. In this regard, the effects of area, depth and position of treatment relative to existing building structure on the performance are particularly important.

Further, ground reinforcement using vertical stone columns involves larger settlements which may be intolerable for structures. This requires that relatively stiffer treatment with cemented columns be adopted for structures to meet the tolerable settlement limits [8], which need further study. In addition, stone columns installation causes vibration-induced settlements. Treatment with inclined micropiles under shallow foundations of existing buildings could not reduce settlements to tolerable limits. This requires that study on vertical type treatment be carried out [5-8] to adopt it for shallow foundations of existing buildings.

This study is the part of comprehensive research carried out on jet grouted columns applied as liquefaction remediation technique to mitigate the damages to the foundation of structure during earthquakes at Nottingham Centre for Geomechanics.

The scope of the research presented in this paper is limited to the study of vertical type jet grouted columns applied around the footing of existing building structures to mitigate liquefaction and to reduce related settlements.

2. THE CASE CONSIDERED FOR THE STUDY

One typical case of building on shallow foundation, founded on the natural ground with design parameters given below was evaluated.

2.1 The Structure Founded on the Soil Deposit

For this study, the isolated footing of 4x4m pad size and 1m thickness was constructed at 1m depth from the ground surface. This footing supports a central column of 1x0.5m cross-section and 5m in length, which supports a part of superstructure, as shown in Fig. 1.

The soil profile with two layers shown in Fig. 2 was taken for this study. This soil deposit consists of liquefiable medium dense (LB) E-Fraction Leighton Buzzard silty sand layer with thickness of 10m as the surface layer (at 40% relative density) underlying which non liquefiable dense layer (at relative density of 80%) with thickness of 10m. Further details for structure, its properties and soil profile have been described in detail in Almani, [9].

3. NUMERICAL MODELLING CODE SELECTION AND COUPLING OF MODULES

The liquefaction and its mitigation can be modelled and analyzed by numerous codes. The degree of coupling in these codes range from full to partial coupling.

The computer code FLAC 2D Version 6.0 was chosen for numerical analysis. Liquefaction problem can be modelled in this partially-coupled solution code by coupling the dynamic module with ground water flow module. For more details see the FLAC User's Manuals [10].

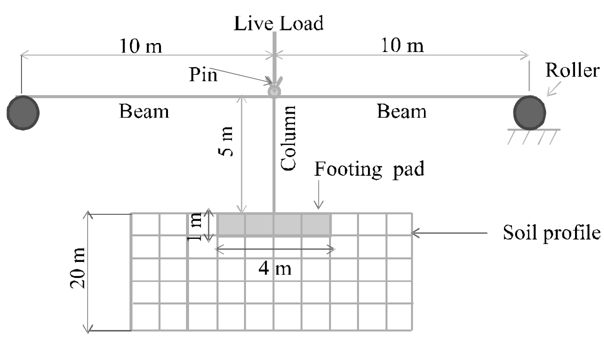

FIG. 1. SUPERSTRUCTURE AND FOOTING OF 4m WIDE X 1m DEEP (MESH AND STRUCTURE NOT DRAWN AS PER SCALE BUT SHOWS MODEL QUALITATIVELY)

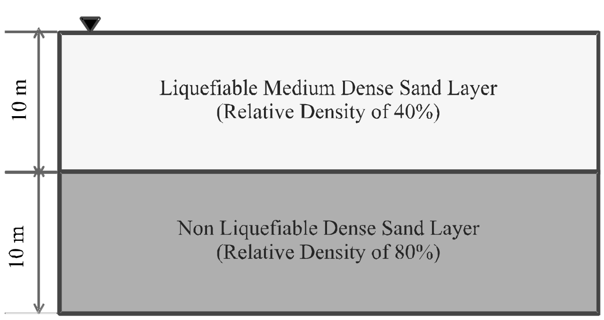

FIG. 2. THE SOIL PROFILE

3.1 Earthquake Excitation

For obtaining consistent results from the sensitivity studies, the sinusoidal horizontal velocity wave with the amplitude of 0.2 m/sec was applied at the bottom of model. This design earthquake simulates a heavy earthquake with a horizontal component of ground acceleration of 0.35g (approximately 3.5 m/sec^2) for duration of 10 sec. The simulation was also continued for a few seconds after the wave stops at 10 sec to bring the model in static equilibrium condition.

3.2 Basic Soil Properties, Soil Liquefaction Model and Damping

The Finn/Byrne liquefaction soil model in FLAC 2D for modelling the phenomenon of liquefaction is based upon Mohr-Coulomb failure criteria in conjunction with hysteric model [10]. This combined constitutive model presents the liquefaction behaviour of soil which is used for this analysis. With this model, decrease of shear modulus and increase of damping ratio with strain level during dynamic simulation occur as per modulus reduction graph is given by Seed, et. al. [11].

The soil elastic properties shear and bulk modulus were calculated from the Hardin/Drnevich equations used for sandy soils [12]. The elastic shear and bulk modulus of the two layers of soil were taken as varying from ground surface which is the function of confining pressure.

The plastic property like drained peak friction angle was measured using consolidated drained triaxial test [13]. The Constant head permeability test was used to measure the permeability of soil. The relationship between shear and volumetric strain is defined by Finn/Byrne model with model parameters C_1 and C_2, therefore the dilation angle which also express the same characteristics was taken as zero. The soil properties are given in Table 1.

For determination of model parameters, the Finn/Byrne soil model was calibrated by simulating the constant volume cyclic simple shear tests in FLAC 2D as single element test as described in detail in Almani, [9]. Then trial and error was performed to get the best values of the soil model parameters C_1 and C_2 and the Hardin-Drnevich hysteretic damping strain constant (Δ_{ref}) as shown in Table 2 by matching FLAC 2D simulated single element simple shear test with laboratory simple shear test. These values were used as model parameters in the Finn/Byrne equation [8] as given in Equation (1):

$$\Delta\varepsilon_V = C_1 \exp\left(\frac{C_2 \varepsilon_V}{\gamma}\right) \tag{1}$$

In Equation (1), $\Delta\varepsilon_v$ is volumetric strain increment in each cycle, ε_v is the accumulated volumetric strain from previous cycle and γ is the shear strain for the cycle, C_1 and C_2 are soil model constants. Hardin-Drnevich hysteretic damping strain constant (Δ_{ref}) for the Hardin-Drnevich hysteretic dynamic damping model was taken as 0.05 for sand, as recommended in the FLAC 2D manual [10].

TABLE 1. PROPERTIES OF SOIL LAYERS

Property	Soil Layers	
	Medium Dense Layer	Dense Layer
Relative Density	40%	80%
Unit Weight (KN/m^3)	18.80	19.57
Porosity (Void Ratio)	0.47 (0.88)	0.42 (0.72)
Permeability (m/sec)	$2\text{x}10^{-7}$	$1\text{x}10^{-7}$
Peak Friction Angle (Degree)	32	48
Model Parameters for Pore Pressure	C_1=1.2; C_2=0.33	C_1=0.43; C_2=3.75
Hardin-Drnevich Damping Constant (Δ_{ref})	0.05	0.05
Water Bulk Modulus (kPa)	$5\text{x}10^5$	$5\text{x}10^5$
Water Tension (kN/m)	$1\text{x}10^2$	$1\text{x}10^2$
Water Density (kg/m^3)	1000	1000

TABLE 2. MODEL PARAMETERS

Model Parameters	Medium Dense Sand Layer	Dense Sand Layer
C_1	1.2	0.43
C_2	0.33	3.75
Δ_{ref}	0.05	0.05

3.3 Ground Reinforcement

The ground was reinforced with stiff jet grouted/deep mixing circular column rows of a relatively same small diameter of 0.6m (or 0.5x0.5m square columns with cross-sectional area equal to circular columns). The length of columns was 11m from the base of footing (9m in surface liquefiable layer and 2m in underlying non liquefiable dense layer). These column rows are provided in different geometric arrangements. In one pattern as commonly used [14], continuously overlapping column rows are provided, as walls (parallel to the sides of the footing in one direction) and as a lattice (parallel to the sides of footing in both directions). In the other pattern, discrete rows of columns are provided (as used under the shopping centre site in Turkey described by Martin, et. al. [4]. The column jet grouted material (cemented sand) was represented with the Mohr-Coulomb soil model combined with Hardin-Drnevich hysteretic dynamic damping model during dynamic analysis. This is done to incorporate the reduction of shear modulus and increase of damping ratio with strain level. Higher initial shear modulus and lower damping ratio of cemented jet grouted sand was taken than uncemented sand. The variation pattern of shear modulus and damping ratio was taken as the same in both sands because it in narrow band as found in cyclic tests [15]. The input model parameters such as Elastic, Shear, and Bulk modulus; plastic properties such as cohesion and tensile strength were determined in the laboratory are shown in Table 3. As columns are located below the water table, they were saturated like the soil.

4. TOLERABLE MOVEMENT CRITERIA OF THE BUILDING STRUCTURE

The settlements tolerable to the building structures in the event of earthquake given in building codes depend upon the type of building, nature of its components, functional use and dimensions. In view of this, settlement criteria with one specified limits is not recommended in building codes. For a typical case of building in this study, settlements for the structure were quantified in tolerable limits based on the recommendations given in literature.

(i) Tolerable limits of 4cm recommended by Skempton and MacDonald [16]. These are the design limits for maximum settlements up to which building is in serviceable condition.

(ii) Tolerable limits of 5cm recommended by European Committee for Standardization [17]. These are also the design limits for maximum settlements up to which the building is in service condition.

(iii) Limits of 10cm at which the structure loss serviceability but may not collapse [18].

5. MODEL DEVELOPMENT

For this study, the rectangular mesh with uniform zone size of 0.5x0.5m and the aspect ratio of one (1) was made as shown in Fig. 3.

The fixed lateral and bottom boundaries were applied during the static analysis whereas during dynamic analysis (earthquake loading), free field boundaries were applied at the two lateral boundaries of the model. The lateral boundaries were of the model were taken at far distance that the behaviour of the soil-structure system (stresses, strains and pore pressure) in the area of structure and surrounding soil is not influenced due effects of boundaries. In this case the model was made so wide that lateral boundaries are three times of the soil profile depth (60m to the each side of shallow footing centre).

In order to mimic hydraulic boundaries, the pore pressures for the top drainage boundary were set as zero (0) to simulate that drainage surface. The lateral and bottom boundaries of the model were taken as impervious. Further details for model development are explained in Almani, [9].

TABLE 3. PROPERTIES OF JET GROUTED COLUMN MATERIAL

Properties	Values
Saturated Unit Weight (KN/m^3)	19
Shear Modulus, G(kPa)	2x10^6
Bulk Modulus, K(kPa)	2.6x10^6
Unconfined Compressive Strength at 28 Days Curing Time (kPa)	4800
Friction Angle (Degree)	0
Cohesion (kPa)	2400
Water Cement Ratio	1:1
Tensile Strength (kPa)	480
Permeability (m/sec)	1x10^{-8}

6. RESULTS AND DISCUSSION

In this study, the isolated shallow strip footing of the structure was founded on the ground reinforced with rows of grouted columns around the footing pad in following geometric arrangements.

6.1 Ground Reinforcement Adjacent to Footing of Existing Buildings

In the first study, only one row of columns was provided adjacent to each side of the footing pad as shown Fig. 4.

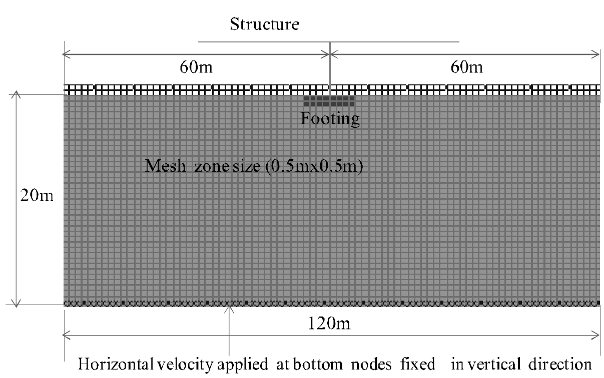

FIG. 3. FINITE DIFFERENCE MESH USED FOR FLAC 2D NUMERICAL ANALYSIS

The results show that the settlement at the centre of the footing pad, as shown in Fig. 5, is within the limits of 10cm as against 85cm in the benchmark (no reinforcement) case described in Almani, [9].

6.2 Effect of Offset from Edge of Footing

In this study, one row and three continuous column rows are provided at the offset distance of 0.5 and 1m from the each edge of the footing pad, as shown in Figs. 6-7.

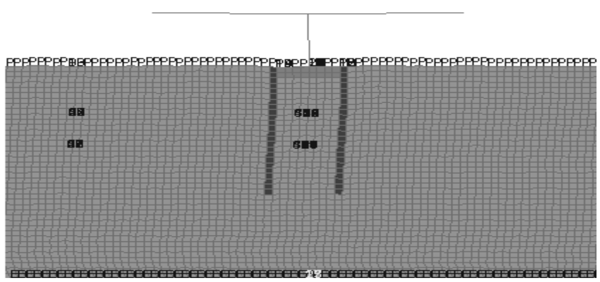

FIG. 4. ONE ROW OF COLUMNS ADJACENT TO PAD

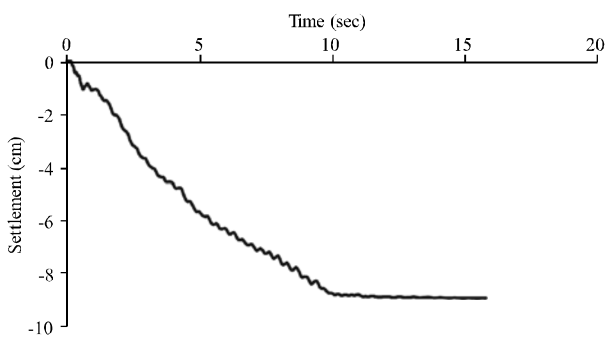

FIG. 5. SETTLEMENT OF FOOTING VS. TIME FOR ONE ADJACENT ROW

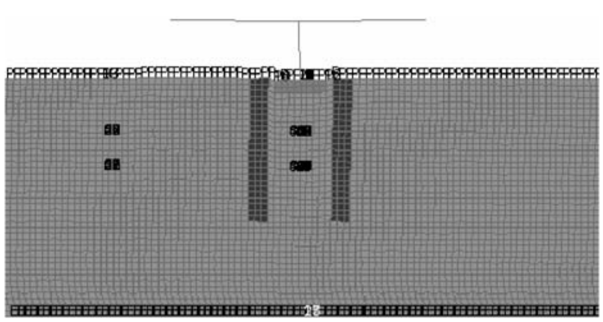

FIG. 6. THREE CONTINUOUS ROWS OF COLUMNS AT OFFSET TO PAD

The treatment performance in this case was compared with the case when one and three continuous rows were provided just adjacent to each pad edge.

The previous results established that when the rows of columns are provided just adjacent to (at zero distance) the edge of the pad, the settlement was within the tolerable limits of 5cm with three continuous rows of columns. This settlement was within the limits of 10cm with one row of columns. The results of this study show that as the offset distance (of one row or three rows) from the edge of the footing increases to 0.5m or larger, the effectiveness of column rows to reduce the settlement decreases drastically. The settlement exceeds the limits of 10cm despite huge treatment of three continuous rows of columns even at a small offset distance of 0.5m from the edge of the footing. The settlement further increases to very large values when the offset distance of rows increases to 1m.

The failure state of the composite ground can be observed by drawing the contours of vertical displacement for the case when three continuous rows were provided at 0.5m from the footing edge, as shown in Fig. 8. This indicates that the footing and the wedge of soil beneath the footing forming a rigid block punches in shear in soft loose liquefied soil which displaces the soil beneath it downward into the narrow zone (without displacing the columns even at the small offset of 0.5m).

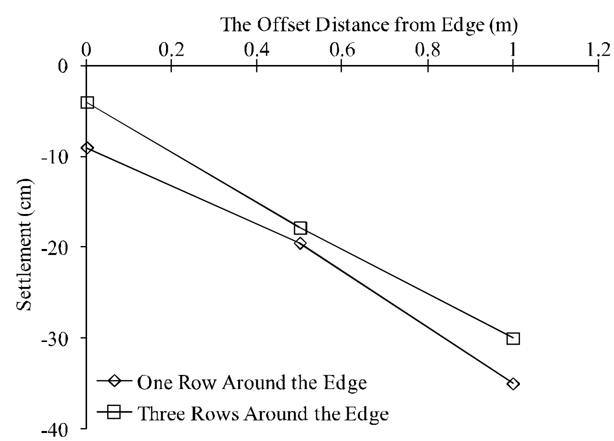

FIG. 7. SETTLEMENT OF FOOTING VERSUS OFFSET DISTANCE FROM PAD EDGE

6.3 Sensitivity Studies for Treatment Optimization

In order to optimize the treatment, the number of continuous rows of jet grouted columns was varied (from one row to eight continuous rows) adjacent to each edge of the footing pad, as shown Fig. 9. Having achieved the effective geometric position of stiff columns for performance of reinforced ground under the structure, sensitivity studies were carried out for determining optimum treatment.

The results show that the settlement at the centre of the footing at the end of cyclic loading is within limits of 10cm when one row of columns is provided adjacent to each footing edge. The settlement decreases to the tolerable limits of 5cm when three continuous rows of columns (each column of 0.5m diameter) are provided adjacent to each footing edge. The settlement further decreases to the tolerable limits of 4cm with further increase in the number of rows of columns to six adjacent to each edge of footing.

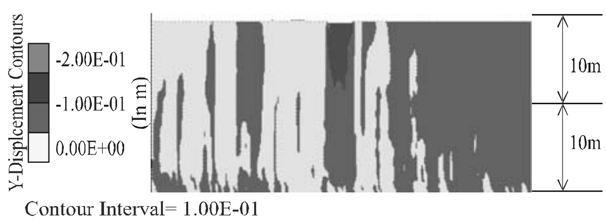

FIG. 8. VERTICAL DISPLACEMENT CONTOURS-THREE CONTINUOUS ROWS AT OFFSET OF 0.5m

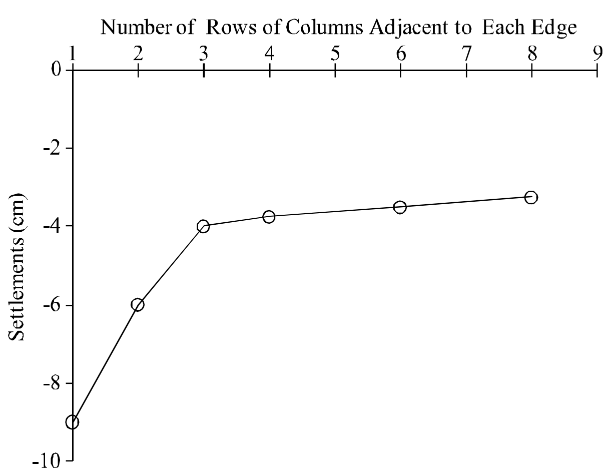

FIG. 9. SETTLEMENT VERSUS NUMBER OF ROWS OF COLUMNS ADJACENT TO THE FOOTING PAD

These results suggest that just one row of columns adjacent to each edge of pad is sufficient to reduce the settlement to 10cm limits. However, three continuous rows of columns adjacent to each edge of the footing pad is the optimum treatment for this specific case to reduce the settlements to meet the tolerable limits of 5cm, though the relative reduction in settlement is low with increase in treatment. For sensitive structures, six rows can be provided to improve further the settlements to meet tolerable limits of 4cm criteria.

The reason for larger settlements when one row of columns was provided adjacent to each edge of the pad can be found from the contours of vertical displacement, as drawn in Fig. 10.

The contours show that the stiff columns punch at their tip level into the underlying non liquefiable dense base layer when the bearing resistance of surface liquefiable layer to support the footing pad and frictional resistance along shaft of columns is lost as result of softening and liquefaction state of the soil. As a result, the soil at the tips of the columns is overstressed when all the load of the structure is transferred to that underlying non liquefied dense layer. Due to this overstress and the punching of columns at their tips into this underlying dense layer, the footing pad with the foundation soil beneath it displaces vertically downward in a block form.

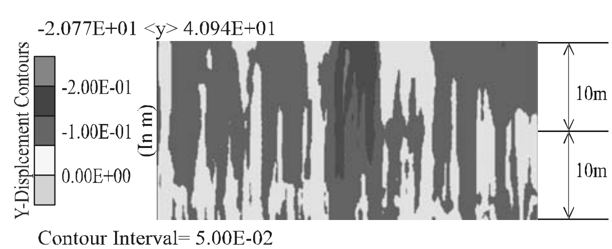

FIG. 10. VERTICAL DISPLACEMENT CONTOURS-ONE ROW ADJACENT TO EACH EDGE OF PAD

The plasticity state of columns, for this case presented in Fig. 11, show that the columns are at a yield (failure) state in tension due to bending caused by the eccentric vertical of structure and the lateral loads from encapsulated soft soil when the surface liquefiable layer is in the liquefied state.

The above results reveal that due to both reasons (the punching of columns in the bottom dense layer and their yielding in tension) the settlements are relatively large with one row. This requires that more continuous rows be provided to the increase x-sectional area of treatment to reduce compressive stresses at the tips and tensile stresses along the shaft due to bending.

6.4 Effect of Multiple Footings

The structures are supported on multiple numbers of footings rather than on one footing. In order to investigate the effect of a multiple number of footings on treatment performance, three footing pads of the same size (4m width and 1m thickness), with one carrying the load of the central interior column and the other two of exterior columns, were modelled and analyzed for existing buildings as shown in Fig. 12.The footing pads were reinforced with three rows (optimum number of rows) each edge of pad.

The results for A Type geometry as presented in Figs. 13-14, show that the settlements at the centre of all (central interior and the exterior) footings and the differential settlements of the left and right exterior relative to the central interior footing are within the tolerable limits of 5cm.

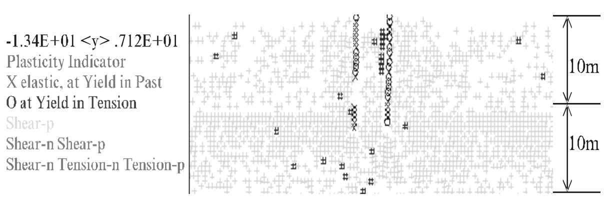

FIG. 11. PLASTICITY STATE OF COLUMNS FOR ONE ROW ADJACENT TO PAD

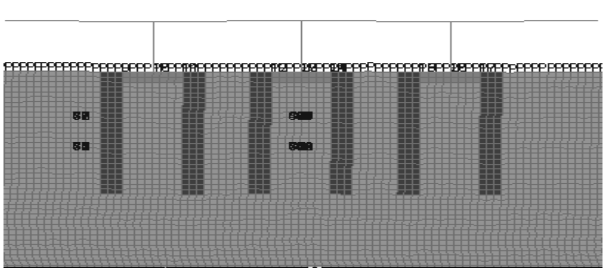

FIG. 12. MULTIPLE FOOTINGS WITH ADJACENT TREATMENT

The above results suggest that the settlements in the case of a multiple number of footings are in the same tolerable limits as in the case of a single isolated footing. On the basis of these results, single footing was taken for design.

7. CONCLUSIONS

The following conclusions can be drawn from the above studies:

(i) The settlement of shallow foundations of buildings can be reduced by providing the number of continuous rows of vertical columns adjacent to each side of the footing of existing buildings.

(ii) The relative benefit decreases as the number of rows increases. When it is imperative to limit the

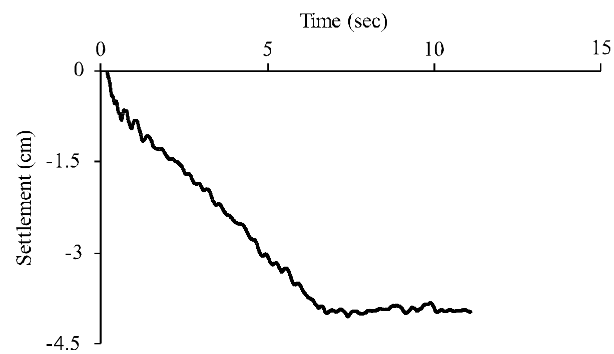

FIG. 13. SETTLEMENT OF LEFT (EXTERIOR) FOOTING VERSUS TIME

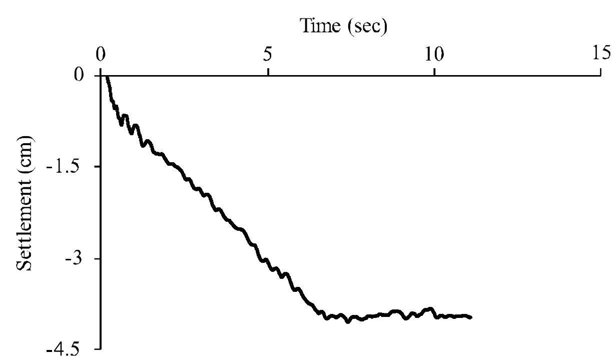

FIG. 14. SETTLEMENT OF CENTRAL (INTERIOR) FOOTING VERSUS TIME

settlements to tolerable limits, three continuous rows of columns (0.5m diameter) adjacent to each edge of the footing pad can be provided for this case as optimum treatment to bring the settlements to tolerable limits of buildings.

(iii) The jet grouted column rows provided at even small offset distance around the footing pad are not effective to improve the settlements to the tolerable limits due to punching of the footing in monotonic shear in the soft liquefied soil in the narrow zone around it. Therefore, continuous rows should be provided adjacent to footing pad of existing buildings.

(iv) Providing the number discrete columns rows around the footing in whole building area at certain spacing to reduce the pore pressures, liquefaction and liquefaction-induced settlements is not effective geometry.

(v) The settlements by analysing multiple number of footings with adjacent treatment is of same order as with single footing. Therefore, the value of settlement for single footing can be taken for design of adjacent treatment for building shallow footings.

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