
Two-Dimensional Stagnation-Point Velocity-Slip Flow and Heat Transfer over Porous Stretching Sheet

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ABSTRACT

Present paper investigates 2D (Two-Dimensional) stagnation-point velocity-slip flow over porous stretching sheet. The governing non-linear PDEs (Partial Differential Equations) are non-dimensionalized by using the similarity transformation technique that results into coupled non-linear ODEs (Ordinary Differential Equations). Such ODEs are then solved by using shooting technique with fourth-order Runge-Kutta method. Since the behavior of boundary layer stagnation-point flow depends on the rate of cooling and stretching. Therefore, the main objective of this paper is to analyze the effects of different working parameters on shear stress, heat transfer, velocity and temperature of fluid. The results revealed that the velocity-slip has significant effect on the fluid flow as well as on the heat transfer. The numerical results are also compared with existing work for no-slip condition and found to have good agreement with improved asymptotic behavior.

Key Words: Stretching Sheet, Similarity Transformation Technique, First-Order Velocity-Slip, Porous Medium, Shooting Technique, Fourth-Order Runge-Kutta Method.

1. INTRODUCTION

Due to inevitable importance in technology and industry, study of boundary layer flow and heat transfer has remained the hot area of research. Few applications are electronic chips, drawing and stretching of plastics films, hot rolling. Such applications are of great interests because the quality of the product depends upon the rate of stretching and cooling. The study has its origin from the work of Sakiadis [1] and Crane [2]. Since then numerous researchers [3-6] have studied it for different kind of flows.

A stagnation-point flow is a flow around a point, stagnation-point, where the velocity of the considered object is zero [7]. Such kind of fluid flows have been of

great interest for researchers due to its tremendous engineering applications including rockets, aircrafts, submarines and oil ships motion where stagnation-point plays very significant role. Due to enormous applications, it has remained focus of many researchers. Steady-state stagnation-point flow over the stretching surface was initially studied by Chiam [8] where he considered same stretching and free stream velocities. His work was extended by Mahapatra and Gupta [9] by setting different stretching and free stream velocities. In past decade, Nandy and Pop [10] taken into consideration the magnetic field and thermal radiation effects on the stagnation-point flow in which they used nanofluid over

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shrinking surface. Stagnation-point flow with heat generation/absorption and convective boundary conditions were studied by Alsaedi, et. al [11]. Attia [12] investigated porosity effects on the stagnation-point flow over stretching surface in the presence of heat generation and absorption. She used finite difference method to study such problem. Her work was improved by Kazem, et. al. [13] and produced converged results by using semi-analytic method called HAM (Homotopy Analysis Method) of Liao [14] and numerical relaxation method of Zwillinger [15]. However, the characteristics of the fluid flow remained same.

Literature depicts that much attention has been paid toward the study of boundary layer fluid flow with no-slip boundary condition but there exists physical problems where such condition is not appropriate. Considering the velocity-slip condition at the surface boundary certainly helps to understand the characteristics of fluid. In this regards, first-order velocity-slip effects on stagnation-point flow over both stretching and shrinking surface was studied by Aman, et. al. [16]. Turkyilmazoglu [17] investigated analytically the magnetohydrodynamic slip flow past a stretching sheet and reported multiple solutions. Rosca and Pop [18] analyzed the effects of second-order velocity-slip over both vertically stretching and shrinking sheet. Dual solutions were obtained by Singh and Chamkha [19] of flow and heat transfer of second-order velocity-slip flow. The flow was considered over vertically permeable shrinking sheet. Aly and Vajravelu [20] studied magnetic and velocity-slip effects of the 2D axisymmetric flow over stretching surface and presented both numerical and exact solution. Hakeem, et. al. [21] took into consideration the second-order slip flow of a nanofluid over both shrinking and stretching sheet under the influence of magnetic and thermal effects. Dual solutions were obtained by Mishra and Singh [22] taken into consideration thermal and velocity-slip flow of a mixed convection flow over permeable shrinking cylinder.

Behavior of fluid flow may be better understood by considering velocity-slip at the boundary. Literature depicts that very little attention has been paid to this problem. Therefore, in the present work, partial-slip effects on the flow and heat transfer of a 2D stagnation-point flow through porous medium is studied. Review of the literature shows that no such study has been done before.

2. MATHEMATICAL FRAMEWORK

Consider steady-state vertical 2D incompressible viscous stagnation-point flow over a stretching sheet. The flow of the fluid is confined to the upper plane $y > 0$. Stretching sheet is stretched linearly along horizontal x -axis in both directions by the equal stretching velocity $u_w(x) = cx$, $c > 0$ being constant. The boundary layer flow moves with the stretching velocity of sheet and the ambient velocity, i.e. velocity of fluid away from stretching sheet, is set as $u_c(x) = ax$, $a > 0$ being constant. According to Darcy flow model, the fluid flows along positive y -axis and strike the surface $y = 0$, as shown in Fig. 1. The fluid flow is divided into two streamlets leaving in positive and negative x -axis respectively.

Thus the governing equations for 2D stagnation-point flow model under the conditions described above are given as [13]:

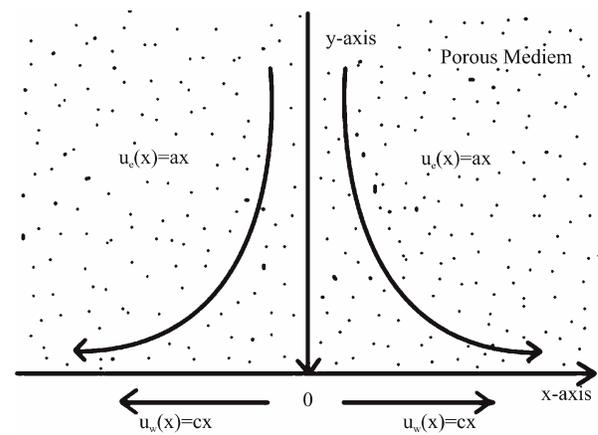


FIG. 1. PICTORIAL REPRESENTATION OF GOVERNING MATHEMATICAL MODEL

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{\partial u_e}{\partial x} + \mathcal{G} \frac{\partial^2 u}{\partial y^2} + \frac{\mathcal{G}}{K} (u_e - u) \quad (2)$$

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + Q(T - T_e) \quad (3)$$

Where u and v , respectively represents the velocities of fluid in x and y direction, ρ is fluid density, K is Darcy permeability, \mathcal{G} is kinematic viscosity, T is fluid temperature, T_e is ambient fluid temperature, Q is rate of heat generation/absorption, k is fluid thermal conductivity and C_p is specific heat capacity at a constant pressure.

Setting the boundary conditions as:

$$\begin{aligned} u &= u_w + U_{\text{slip}}, v = 0, T = T_w \text{ at } y = 0 \\ u &\rightarrow u_e, T \rightarrow T_e \text{ as } y \rightarrow \infty \end{aligned} \quad (4)$$

where U_{slip} is the first-order velocity-slip which was proposed by Maxwell [23] and T_w is the boundary wall temperature. Suitable similarity transformation variables for the above set of Equations (1-4) are:

$$\eta = y \sqrt{\frac{c}{\mathcal{G}}}, f(\eta) = \frac{\psi}{x \sqrt{c \mathcal{G}}}, \theta(\eta) = \frac{T - T_e}{T_w - T_e} \quad (5)$$

Where $f(\eta)$ and $\theta(\eta)$, respectively, are the dimensional stream function and temperature function of independent similarity variable η . Stream function ψ , which identically satisfies Equation (1) and can be checked easily, is defined as $u = \partial \psi / \partial y$ and $v = -\partial \psi / \partial x$. Using Equation (5) one can easily get:

$$u = c x f'(\eta), v = -\sqrt{c \mathcal{G}} f(\eta) \quad (6)$$

where prime represents the ordinary differentiation of the variable with respect to η . Using Equations (5-6) in Equations (2-4), we get:

$$f''' + f f'' - f'^2 + M(C - f') - C^2 = 0 \quad (7)$$

$$\theta'' + \text{Pr} f \theta' + \text{Pr} B \theta = 0 \quad (8)$$

$$f(\eta) = 0, f'(\eta) = 1 + S f''(\eta), \theta(\eta) = 1 \text{ at } \eta = 0$$

$$f'(\eta) \rightarrow C, \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \quad (9)$$

Where M is porosity parameter, C is sheet stretching parameter, Pr is Prandtl number, B is heat generation/absorption coefficient and S is velocity-slip parameter, which are defined as:

$$M = \frac{\mathcal{G}}{cK}, C = \frac{a}{c}, \text{Pr} = \frac{\mu C_p}{k}, B = \frac{Q}{C \rho C_p}, S = L \sqrt{\frac{c}{\mathcal{G}}}$$

3. NUMERICAL PROCEDURE

In order to use Runge-Kutta fourth-order method to solve system of ordinary differential Equations (7-8) boundary conditions Equation (9), it is necessary to find the unknown initial conditions, that are, $f'(0)$ and $\theta'(0)$ such that the given asymptotic boundary conditions remain satisfied. This way given boundary value problem will be transformed into initial value problem and Runge-Kutta fourth-order method can be easily applied. To find such magical values shooting technique [24] has been adopted. For such numerical procedure the domain of the problem was confined to $\eta \in [0, 8]$. The step size of $h = 0.001$ was used and tolerance for truncation of the error was set as 10^{-8} which is sufficient to insure the accuracy of the solution.

4. RESULTS AND DISCUSSION

The numerical study has been done to study the velocity-slip effects on the rate of heat transfer $-\theta'(0)$, variation of shear stress $f''(0)$, temperature distribution $\theta(\eta)$ and fluid velocity $f'(\eta)$. Effects of physical parameters, such as, stretching parameter C , Prandtl number Pr , porosity parameter M , slip parameter S , and dimensionless heat

generation/absorption coefficient B , are analyzed and discussed in details both numerically and graphically. For validation, the numerical results are also compared with the existing numerical results of Kazem, et. al. [13] simultaneously.

Table 1 depicts the effects of velocity-slip on the variation of shear stress $f''(0)$ at different values of slip parameter S , stretching parameter C and porosity parameter M . It

can be observed that the shear stress is significantly influenced by velocity-slip, shear stress increases with increase in slip. Fig. 2 presents the graphical representation of the effects of slip on fluid velocity $f'(\eta)$. It can be stated that the slip affects on the velocity of fluid differently for different stretching parameter value under consideration. Increase in slip produces increase in fluid velocity when value of stretching parameter is greater than 1, and inverse behavior is noted when value of

TABLE 1. SLIP EFFECTS ON THE SHEAR STRESS $f''(0)$ AT DIFFERENT VALUES OF M AND C FOR $B=0$ AND $Pr=0.7$

S	C	Present Solution	Homotopy Analysis Method [13]	Numerical [13]	Present Solution	Homotopy Analysis Method [13]	Numerical [13]	Present Solution	Homotopy Analysis Method [13]	Numerical [13]
		M=0			M=1			M=2		
0	0.5	-0.667264	-0.6675	-0.6672	-0.832126	-0.8321	-0.8321	-0.970103	-0.9702	-0.9701
	1	0	0	0	0	0	0	0	0	0
	1.5	0.909530	0.9097	0.9095	1.036445	1.0365	1.0364	1.149870	1.1499	1.1499
0.5	0.5	-0.386326	-	-	-0.448593	-	-	-0.484031	-	-
	1	0	-	-	0	-	-	0	-	-
	1.5	0.485336	-	-	0.500850	-	-	0.541162	-	-
1	0.5	-0.275331	-	-	-0.305349	-	-	-0.324794	-	-
	1	0	-	-	0	-	-	0	-	-
	1.5	0.328121	-	-	0.341534	-	-	0.351909	-	-
1.5	0.5	-0.214667	-	-	-0.233274	-	-	-0.244759	-	-
	1	0	-	-	0	-	-	0	-	-
	1.5	0.247439	-	-	0.254852	-	-	0.260506	-	-

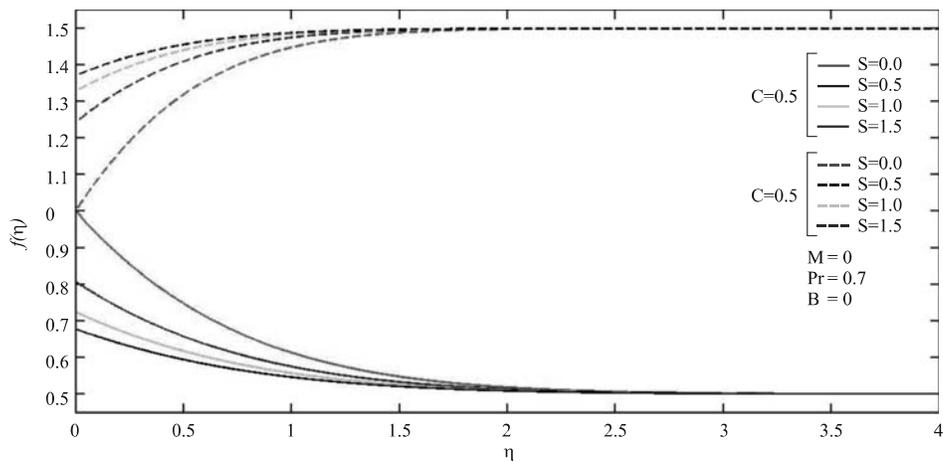


FIG. 2. FLUID VELOCITY $f'(\eta)$ PROFILES FOR VELOCITY SLIP S AND SHEET STRETCHING C

stretching parameter is less than 1. In the absence of velocity-slip, the data for variation of shear stress is compared with Kazem, et. al. [13]. It is found that present results are better than that of Kazem, et. al. [13] solutions as they are more close to the exact solutions. This improvement of solution can also be seen from Figs. 3-5 where the profiles of velocity of fluid satisfies the boundary condition asymptotically. Table 1 also reveals that for the different values of slip and porosity parameter, increase in stretching parameter produces increase in shear stress. Moreover, for the different values of slip and stretching parameter, increase in porosity parameter produces decrease in shear stress.

By considering Figs. 6-7, it can be said that velocity of fluid increases by stretching of surface. Another interesting fact is revealed that when the value of stretching parameter is greater than 1, increase in porosity of medium leads to increase in fluid velocity, whereas inverse behavior is observed when value of stretching parameter is less than 1.

The results are also obtained for the effect of velocity-slip on the wall heat transfer $-\theta'(0)$ at different values of stretching and porosity parameter and are listed in Table 2. It can be observed that the wall heat transfer decreases by increasing the velocity slip parameter for

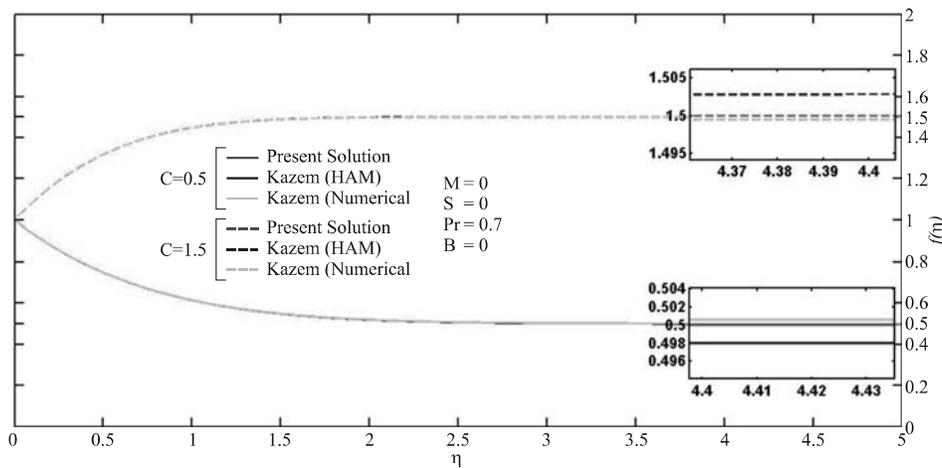


FIG. 3. IMPROVED PROFILES OF FLUID VELOCITY $f'(\eta)$ AT $M=0$

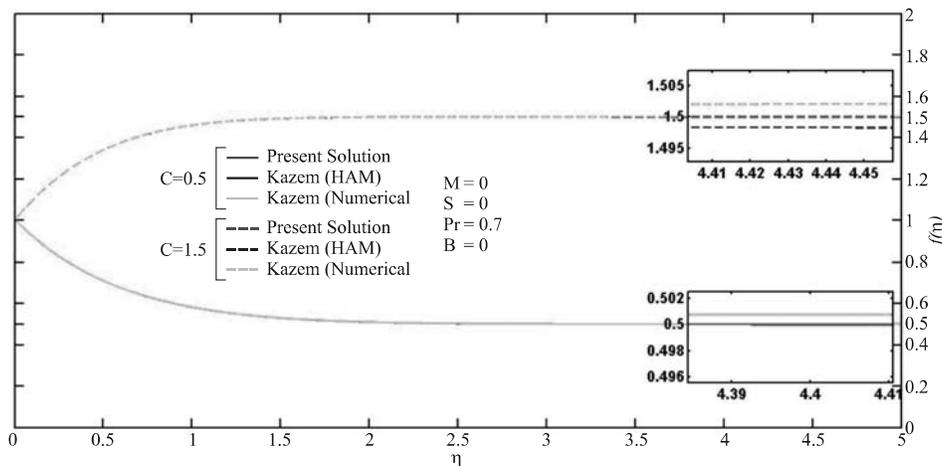


FIG. 4. IMPROVED PROFILES OF FLUID VELOCITY $f'(\eta)$ AT $M=1$

surface stretching less than 1 and increasing for surface stretching greater than 1. In the absence of velocity-slip, the present results are compared with Kazem, et. al. [13].

It can be seen that the present results are also improved as they satisfy the asymptotic condition of boundary. From Table 2, it can also be noted that wall heat transfer

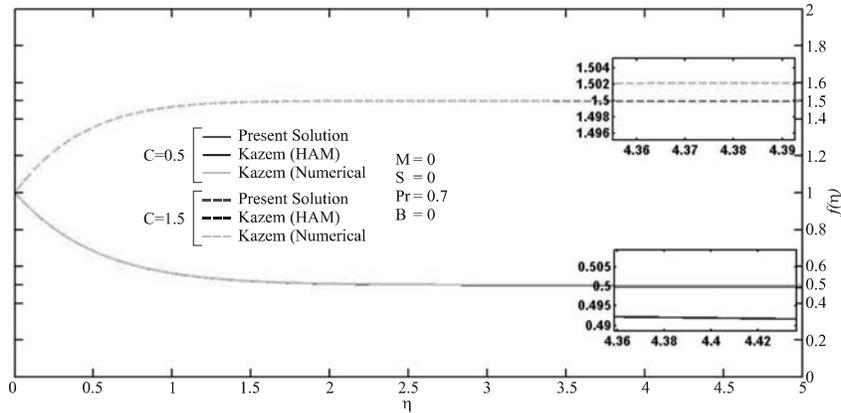


FIG. 5. IMPROVED PROFILES OF FLUID VELOCITY $f'(\eta)$ AT $M=2$

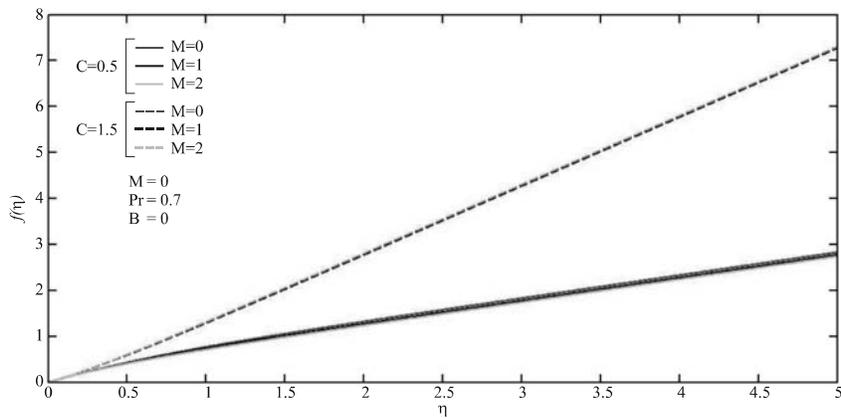


FIG. 6. $f(\eta)$ PROFILES OF $f(\eta)$ FOR MEDIUM POROSITY M AND SHEET STRETCHING C

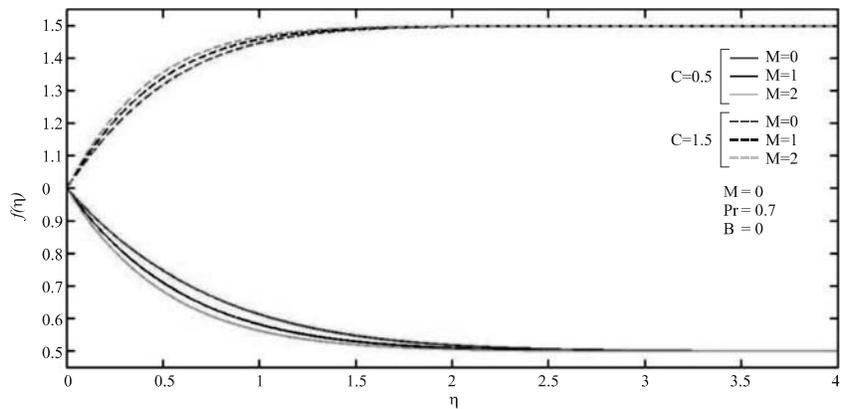


FIG. 7. FLUID VELOCITY $f'(\eta)$ PROFILES FOR MEDIUM POROSITY M AND SHEET STRETCHING C

increases by increase in stretching parameter for different values of slip and porosity parameter. Moreover, it decreases by increase in porosity parameter for different values of slip when surface stretching parameter is less than 1 and increases when greater than 1.

Under no-slip condition, Tables 3-4 present the variation in wall heat transfer $-\theta'(0)$ and comparison with

previously published data of Kazem, et. al. [13]. Present results are improved which can be seen from asymptotical property of graphical results. It can be observed that by increasing the surface stretching and Prandtl number the increase in wall heat transfer rate occurs. On the other hand, increase in porosity of medium and heat generation decreases it.

TABLE 2. SLIP EFFECTS ON WALL HEAT TRANSFER $-\theta'(0)$ AT DIFFERENT VALUES OF C AND M WHEN Pr=0.7 AND B=0

S	C	Present Solution	Kazem, et. al. [13]	Present Solution	Kazem, et. al. [13]	Present Solution	Kazem, et. al. [13]
		M=0		M=1		M=2	
0	0.5	0.569759	0.5691	0.559041	0.5577	0.551545	0.5508
	1	0.667577	0.6677	0.667577	0.6676	0.667577	0.6677
	1.5	0.753695	0.7537	0.757695	0.7578	0.760901	0.7609
0.5	0.5	0.534504	-	0.515769	-	0.514553	-
	1	0.667577	-	0.667577	-	0.667577	-
	1.5	0.785321	-	0.781159	-	0.791943	-
1	0.5	0.518825	-	0.507958	-	0.501570	-
	1	0.667577	-	0.667577	-	0.667577	-
	1.5	0.796174	-	0.799006	-	0.801103	-
1.5	0.5	0.509719	-	0.500209	-	0.494859	-
	1	0.667577	-	0.667577	-	0.667577	-
	1.5	0.801587	-	0.803818	-	0.805454	-

TABLE 3. VARIATION IN WALL HEAT TRANSFER RATE $-\theta'(0)$ AT DIFFERENT VALUES OF C AND Pr WHEN S=0, B=0 AND M=0.5

Pr	Present Solution	Kazem, et. al. [13]						
	C=0.1		C=0.5		C=1		C=1.5	
0.1	0.23809	0.1549	0.25625	0.2004	0.28473	0.2528	0.29074	0.2623
0.5	0.40464	0.3828	0.47587	0.4728	0.56442	0.5642	0.58082	0.5807
1	0.60482	0.6022	0.69252	0.6925	0.79788	0.7979	0.81762	0.8177

TABLE 4. VARIATION IN WALL HEAT TRANSFER RATE $-\theta'(0)$ AT DIFFERENT VALUES OF C AND B WHEN S=0, M=0 AND Pr=0.7

B	Present Solution	Kazem, et. al. [13]						
	C=0.1		C=0.5		C=1		C=1.5	
-0.1	0.53734	0.5305	0.61953	0.6192	0.71257	0.7127	0.79464	0.7948
0	0.46581	0.4530	0.56386	0.5632	0.66758	0.6676	0.75582	0.7558
0.1	0.38343	0.3580	0.50350	0.5024	0.62029	0.6201	0.71558	0.7155

The temperature of fluid is also influenced by velocity-slip. By decreasing the slip temperature is increases when the value of stretching parameter is greater than 1, and temperature decreases by decrease in slip when the value of stretching parameter is less than 1 as shown in Fig. 8. The fluid temperature behavior under different surface stretching, porosity of medium, Prandtl number and heat generation can be seen from Figs. 9-11 respectively. Temperature of the fluid decreases by increase in Prandtl number and surface stretching parameter (Figs. 9-10), whereas it increases with the increase in heat generation parameter (Fig. 11).

5. CONCLUSIONS

Steady-state 2D stagnation-point flow was investigated for effects of velocity-slip over porous stretching sheet. Well know shooting technique combined with fourth-order Runge-Kutta method have been adopted to obtain the solution. Under no-slip case the present results are compared with existing literature and noticed that present results are comparatively better. From the results obtained we conclude that under second:

- (i) The shear stress $f''(0)$ is directly proportional to sheet stretching C , that is, by decreasing/increasing C we notice decrease/increase in $f''(0)$.

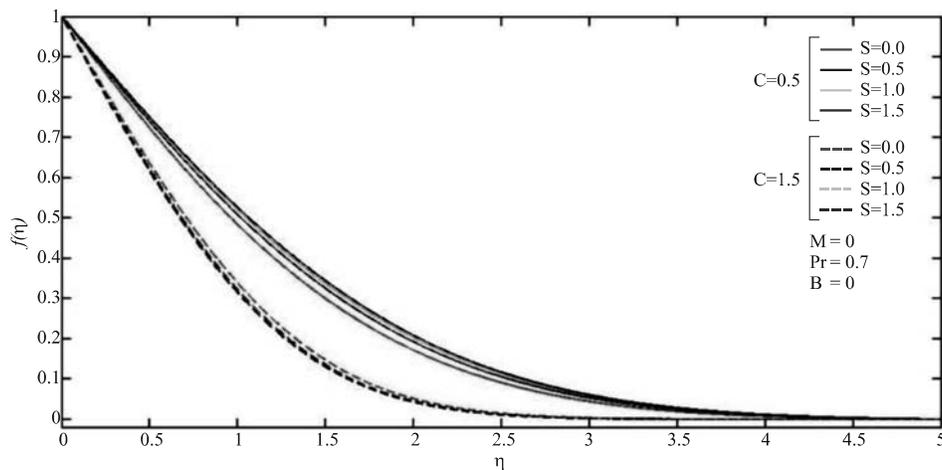


FIG. 8. FLUID TEMPERATURE $\theta(\eta)$ PROFILES FOR BOUNDARY SLIP S AND SHEET STRETCHING C

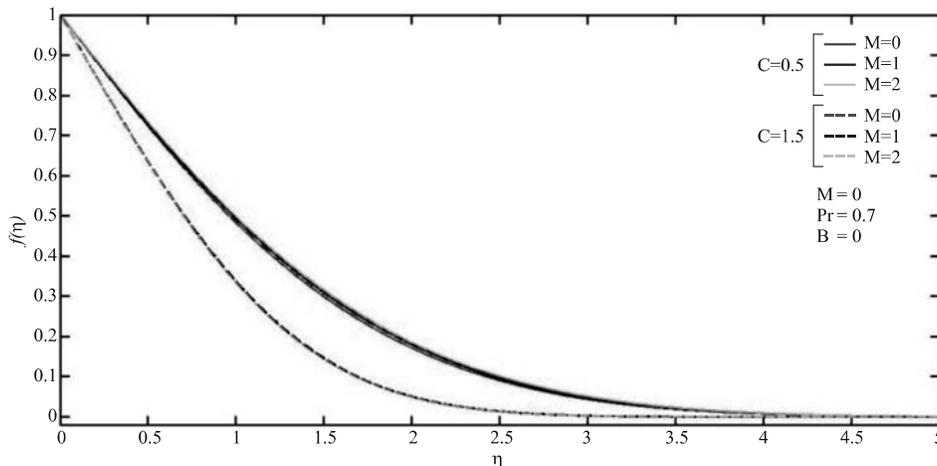


FIG. 9. FLUID TEMPERATURE $\theta(\eta)$ PROFILES FOR MEDIUM POROSITY M AND SHEET STRETCHING C

For all values of porosity of medium M , the shear stress is negative when the surface stretching parameter value is less than 1 and positive when greater than 1.

- (ii) Increasing the boundary slip S decreases the fluid velocity $f'(\eta)$ and temperature $q(\eta)$ when surface stretching parameter value is less than 1 and inverse profile is observed when it is greater than 1.
- (iii) Boundary layer thickness decreases by increase in boundary slip parameter S , sheet stretching parameter C and Prandtl number Pr .
- (iv) For all values of porosity parameter M , increasing sheet stretching C decreases heat transfer $q(\eta)$.

- (v) Temperature of the fluid $q(\eta)$ decreases by increase in Prandtl number Pr and sheet stretching parameter C .
- (vi) For all values of heat generation/absorption parameter B , the fluid temperature $q(\eta)$ decreases/increases by the decrease/increase in sheet stretching C .
- (vii) Temperature for heat absorption is greater than heat generation.

More numerical investigations on stagnation-point fluid flow and heat transfer over stretching sheet under second-order velocity-slip condition will be made in the future work by using shooting technique and other higher-accuracy numerical methods.

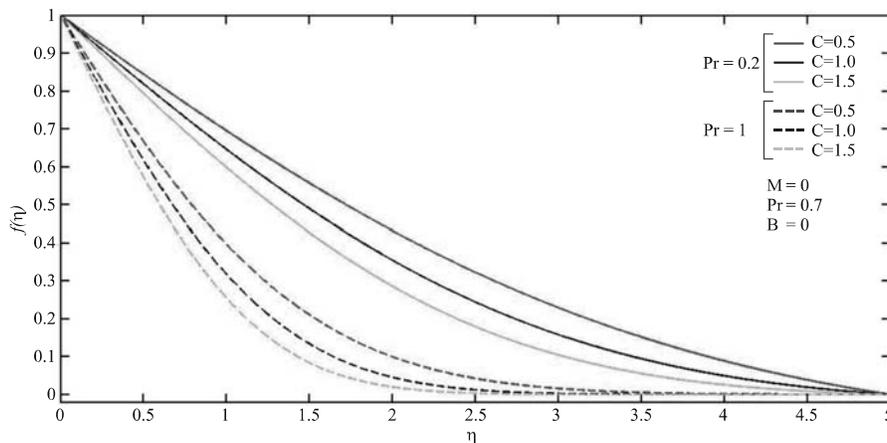


FIG.10. FLUID TEMPERATURE $\theta(\eta)$ PROFILES FOR PRANDTL NUMBER Pr AND SHEET STRETCHING C

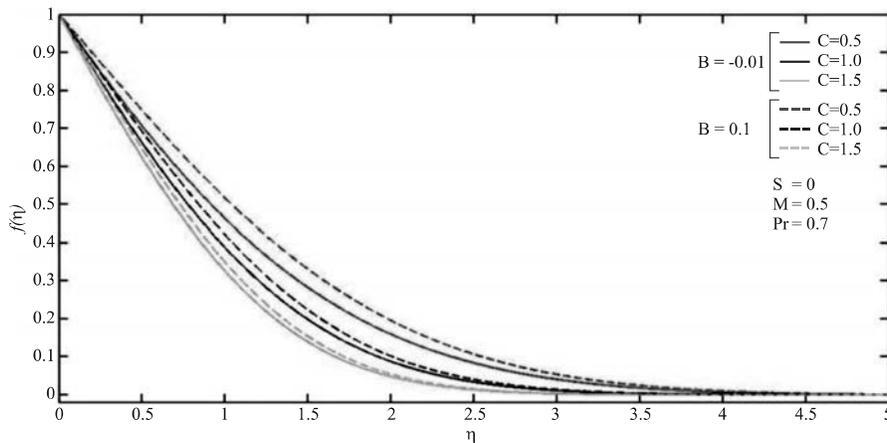


FIG. 11. FLUID TEMPERATURE $\theta(\eta)$ PROFILES FOR HEAT GENERATION/ABSORPTION B AND SHEET STRETCHING C

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