A Hybrid Cuckoo Algorithm for Lot Scheduling Problem Using Extended Basic Period and Power of Two Policy

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ABSTRACT

This paper addresses ELSP (Economic Lot Scheduling Problem) using EBPA (Extended Basic Period Approach) model with PoT (Power of Two) policy. The objective is to solve the ELSP using HCA (Hybrid Cuckoo Search Algorithm). The proposed approach improves the solution (i.e. minimizes the total cost which is the sum of setup and inventory holding costs) obtained through GA (Genetic Algorithm).The solution obtained from HCA is compared with GA on 17 Bomberger's problems. The comparison indicates the superiority of the proposed HCA over GA with respect to the solution quality.

Key Words: Economic Lot Scheduling Problem, Basic Period Approach, Extended Basic Period Approach, Hybrid Cuckoo Search Algorithm.

1. INTRODUCTION

The purpose of ELSP is to find the cyclic solution for production of multiple products on a single production facility. ELSP is under study since 1950 [1-2] and extensive research has been done to find the optimal solution of the problem. Many comprehensive studies have been performed by various researchers under different configurations (i.e. after applying various simplifications and restrictions) to better understand the complexity of the ELSP [3]. These studies proved that ELSP is an NP-Hard problem which means that it is not possible to find an optimal solution of the ELSP (i.e. with or without relaxing the actual problem) using analytical techniques [4-5]. Therefore, ELSP is usually solved using one of the four approaches which include CCA (Common Cycle Approach) [6], BPA (Basic Period Approach) [7-8], EBPA [10] and TVA (Time Varying Approach) [9]. Each of these approaches has its own advantages and disadvantages, but all of these are NP-Hard [4-5] and due to this we would not be able to find an optimum solution using analytical techniques.

In order to solve ELSP many research studies adopted meta-heuristic and nature inspired techniques to solve ELSP [7-8,10-18]. It has been proved that these techniques are quite successful in finding solutions closed to the lower bound solution. Meta-heuristic and nature inspired techniques applied to date includes GA [8,10,12,14], DHS (Discrete Harmony Search) [15], TS (Tabu Search) [16], CS (Cuckoo Search) [7], SA (Simulated Annealing) [7] and PSO (Particle Swarm Optimization) [7]. However, most of the researchers used GA to solve ELSP using BPA [7-8], EBPA [14,16,19] and TVA [9].

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Among BPA, EBPA and CCA approaches, CCA is the simplest one which provides the same cycle time T for all products but the deviation from TCL (Tight Lower Bound) solution is quite high for the same TCIS (Total Cost Independent Solution). BPA relaxes the same cycle time condition of CCA for all products by providing each product with different cycle time which is integer multiple of some basic period T and gives a better result than CCA. The basic period needs to be large enough to accommodate all products which result in still large deviation from TCL. EBPA further relaxes BPA by providing flexibility of not producing each product in every basic period. Therefore, it results in a reduced basic period and due to this it improves the results by further decreasing deviation from TCL. TVA is the most flexible but complex approach to solve ELSP as it provides provision of associating each product with different cycle time for each production cycle of the product. Due to the flexible association of cycle time in TVA it results in reduced deviation from TCL. However, it is worth to mention here that multiple research studies [14,15,19] proved that we can use EBPA with different heuristics to find competitive results of ELSP for both low and high utilization problem without dealing with the complexity of TVA.

This research uses the nature inspired algorithm to solve ELSP problem using EBPA with PoT policy. We have applied HCA (EBPA) to find the solution and compared with existing GA (BPA) [8] and HCA (BPA) [7] based best- known ELSP solutions on Bomberger's dataset [20]. Table 1 shows the detail comparison between the working of GA and HCA. The result obtained through detailed study shows the effectiveness of HCA (EBPA) on low and high machine utilization cases over other existing ELSP algorithms on six benchmark ELSP problems.

TABLE 1. DETAILED COMPARISON OF THE WORKING OF GA AND CSA

Step-1: Create N (i.e., population size) seed solution (i.e., chromosomes and each chromosome represents a solution) that satisfies all the constraints applicable to the problem.	Step-1:Create N (i.e. total host nests) seed solution (i.e., each nest represents a solution) that satisfies all the constraints applicable to the problem.
Step-2: Repeat the following steps till the maximum steps or stopping criterion;	Step-2: Repeat the following steps till the maximum steps or stopping criterion;
Step-3: Create N new population members (i.e., chromosomes) by applying cross over operator to the population members forwarded from the previous generation.	Step-3: Create N new nests by doing Lévy walk around existing best solution found till now (i.e. obtained from previous generation nests).
Step-4: Create M new population members by selecting M random members (i.e., chromosomes) from the population members forwarded from the previous generate and mutate it by applying mutation operator.	Step-4: A host can determine a strange egg with a probability pi. If the pib > pa (i.e., pa is the probability of discovering alien eggs) then the host bird abandon the nest. For all abandoned nests, it creates new nests having locations far away from the existing best solution.
Step-5: Select N best population members (i.e., chromosomes) from N population members from the previous generation, N population members from the current generation created using cross over operator, and M population members from the current generation created using mutation operator.	Step-5: Select N best nests from N previous generation nests and N current generation nests.
Step-6: To keep the number of members in the population same we will only select N population members for the next generation having best values of the objective function and subject to meeting all applicable constraints.	Step-6: To keep the number of nest same we will only select N nests for next generation having best values of the objective function and subject to meeting all applicable constraints
Step-7: The above comparison is based on the basic working of the GA and CSA. However, the actual implementation may require modification in the implementation. This modification may be due to specific requirements of a particular problem, availability of the solution specific heuristic, or to enhance the convergence speed of the algorithm.	

The rest of the paper is organized as follows: Section 2 outlines the problem statement. Section 3 describes the EBP method with PoT policy. Section 4 describes the proposed hybridization approach to solve the EBP with PoT policy. In The detail comparison between GA and CSA is performed in Section 5. Section 6, we compare the results of our proposed technique with GA (BPA) [20] and HCA (BPA) results. We present our discussion and conclusion in Section 7.

2. PROBLEM STATEMENT

ELSP is about the production scheduling of several different items on a single machine on a repetitive basis. The machine can only produce one item at a time with a different production rate for each item. Each item has its own demand rate (i.e. demand is constant for infinite period and shortage is not allowed), setup cost and setup time.

A feasible production schedule is defined as the one in which: (a) at most one item is produced by the facility at any time (b) the total time load on the facility does not exceed the available time capacity; and (c) demand is satisfied without shortages. An optimal solution is the best feasible solution which minimizes the total production cost of all items.

3. EBP MODEL WITH POT POLICY

Facility such that it minimizes the total production cost. EBP solve the production scheduling problem (i.e. finding optimal solution) using the following assumptions.

- There is no precedence of any product over others.
- There is no provision of Back-orders.
- Each item is only produced when its inventory becomes zero.
- The production capability is in perfect condition (i.e. no failure of a machine during production) and produced items have perfect quality (i.e. no product with poor quality).
- In an EBP model a complete production cycle consists of multiple fundamental cycles. It is not required to produce all items in each fundamental Cycle T. However, in order to meet the demand, each item must be produced at least once and at most as many times as the total number of fundamental cycles consists in a complete production life cycle.
- In PoT policy, the cycle time T_i of each item i is an integer multiple of some k_i(i.e. here k_i can only have Power of Two value, like $k_i \in \{1,2,4,8,\ldots\}$)and fundamental cycle T.

4. PROPOSED HCA TO SOLVE ELSP USING EBP MODEL WITH POT POLICY

In this section we will explain the detailed working of our proposed HCA to solve the ELSP using EBP model with PoT policy.

First of all, we need to describe the notations used in the model:

i	:	An item index, i={1,2, …,n}
k_i	:	Integer multiplier of product I, $k_i \in \{1,2,4,8,\ldots\}$
D_i	:	Yearly demand of each item i
P_i	:	Yearly production of each item i
H_i	:	Cost of holding each item i
S_i	:	Cost to setup each item i
τ_i	:	Time required to setup each item i
Q_i	:	Quantity produce of each item i
T	:	Total fundamental cycle time
T_i	:	Time allotted to each item i
TC_i	:	Cumulative yearly cost for holding and setup of each item i
TC	:	Cumulative yearly cost for holding and setup of all items
N	:	Total number of Cuckoo nests
L	:	Total number of fundamental cycles
F_l	:	Products produced in fundamental cycle l, 1<= l<=L
J	:	Production position, $J = \{J_1, J_2, \ldots, J_L\}$

The EBP model for ELSP is given below:

Objective Function,

$$\text{Minimized TCEBP} = \sum_{l=1}^{L}\sum_{i=1}^{n}\left(Tk_i D_i\left(1-\frac{D_i}{P_i}\right)\frac{H_i}{2}+\frac{S_i}{Tk_i}\right)Z_{il} \qquad (1)$$

Where,

The complete production cycle consists of L fundamental cycles.

$$L = \max(k_i) \qquad (2)$$

The value of Z_{il} becomes '1' when item i is produced in a fundamental cycle l (i.e. the value of l is between 1 and L) otherwise its value becomes '0'.

$$Z_{il} \in \{0,1\} \qquad (3)$$

Subject to,

The constraint ensures that the products assigned to each fundamental cycle have enough room to produce all of them.

$$\sum_{i=1}^{n}\left(\frac{D_i}{P}+\frac{S_i}{Tk_i}\right)Z_{il} \le 1,\ 1 \le l \le L \qquad (4)$$

$$J'_i = (l-1)\,(\text{mod}\ k_i) + 1 \qquad (5)$$

Where

$i = 1,2,\ldots n$ and $l = 1,2,\ldots L$

$F_l = \{J_i \equiv J'_i\}$

Where $i = 1,2,\ldots,n$; $l = 1,2,\ldots L$ and J_i is calculated through HCA

The proposed HCA algorithm used to solve the ELSP using EBP model is described below:

- Equation (1) is a non-linear objective function which we need to minimize under the constraint mentioned in the Equation (4)
- The algorithm first need to determine the valid solution bound (i.e. upper and lower) of T and k_i's for the given dataset as discussed in [7-8].

4.1 Seed Solution Generation

Step-1: Initializes k_i's randomly between $[k_i^{LB}, k_i^{UB}]$, $i = 1,2,\ldots,n$ as discussed in [7].

Step-2: Convert the minimum and maximum bound of k_i into the nearest Power of Two (i.e. PoT policy).

Step-3: Create N (i.e. Total Cuckoo nest) initial solutions by randomly generating the value of k_i's between the allowed bounds.

Step-4: Convert k_i's for each of N nest into the nearest Power of Two within the allowed bounds.

Step-5: Compute L for each of N nest using Equation (2).

Step-6: For each N nest, calculate the production position J (i.e. $J = \{J_1, J_2, \ldots, J_L\}$)using Cuckoo search. Compare each value of J with values computed using Equation (5) and only the products satisfying the constraint in Equation (6) will be produced in fundamental cycle l (i.e. F_l, the set containing all product produced in fundamental cycle l).

Step-7: The value of Z_{il} for each product i and each fundamental cycle l will then be determined using F_l calculated in the previous step. (i.e. Z_{ij} value is either '0' or '1'. If $Z_{12} = 1$then it means product 1 is produced in the fundamental cycle 2. Each product can be produced in multiple fundamental cycles. There are L fundamental cycles in a complete production cycle and during each fundamental cycle multiple products can be produced).

Step-8: Given the initial k_i's, the TCEBP subject to constraint Equation (4) can be minimized by performing one dimensional search on fundamental cycle T based on GSS as discussed in [7,13].

4.2 Finding Optimum Solution using Hybrid Cuckoo Search Algorithm

Do the following steps until either the output converged to a particular solution or the iteration reached its maximum limit:

Step-1: First update the value of k_i's associated with each N nests using CSA as mentioned in [7,21-22]. Each nest has a total N number of k_i's (i.e.,k_1, k_2, …, k_N) which are the integer multiplier of fundamental cycle T.

Step-2: For each nest, the k_i's outside the allowed limit of [k_i^{LB}, k_i^{UB}] will be randomly assigned values from the allowed limit.

Step-3: Convert k_i's for each of N nest into the nearest Power of Two within the allowed bounds.

Step-4: Create M (i.e. new Cuckoo nest) new solutions by randomly generating the value of k_i's between the allowed bounds.

Step-5: Convert k_i's for each of N nest into the nearest Power of Two within the allowed bounds.

Step-6: Given newly generated M nest and the updated N nest having k_i's associated with each N+M total nests in k-dimensional search space.

Step-7: Compute L for each of N+M nest using the Equation (2).

Step-8: For each N+M nest, calculate the production position J (i.e. J = {J_1, J_2, …, J_L}) using Cuckoo search. Compare each value of J with values computed using Equation (5) and only product satisfying the constraint in Equation (6) will produced in fundamental cycle l (i.e. F_l, the set containing all product produced in fundamental cycle l)

Step-9: Apply GSS as discussed in [7,13] to find the value of the fundamental cycle T which minimizes TCEBP under the constraint mentioned in the Equation (4).

Step-10: Update current best k_i's and T that minimize TCEBP

Step-11: Update the list of nests by selecting only N best nest out of N+M total nests

The demonstration of the generation of three product solution

Let n = 3 (Total number of products)

Let L = 4 (Computed using Equation (2))

Let $n_1 = 1$ and $J_1 = 1$, (Determine by HCA)

$n_2 = 2$ and $J_2 = 2$, (Determine by HCA)

$n_3 = 4$ and $J_3 = 3$. (Determine by HCA)

Then,

$F_1 = \{1\}$, $F_2 = \{1, 2\}$, $F_3 = \{1, 3\}$, and $F_4 = \{1, 2\}$

And,

For i = 1; $Z_{11} = 1$, $Z_{12} = 1$, $Z_{13} = 1$, and $Z_{14} = 1$

For i = 2; $Z_{21} = 0$, $Z_{22} = 1$, $Z_{23} = 0$, and $Z_{24} = 1$

For i = 3; $Z_{31} = 0$, $Z_{32} = 0$, $Z_{33} = 1$, and $Z_{34} = 0$

4.3 Comparison between GA and CSA Algorithm

To find the optimum (i.e. either minimum or maximum) solution of the problem. An objective function is provided along with a set of constraints (i.e. if the problem is unconstraint than the constraints set will becomes null) that must meet by any solution of the problem.

5. RESULTS

The results obtained through HCA on both BPA and EBPA are shown in Tables 3-4. Table 3 shows the total annual cost using GA (BPA), HCA (BPA) and HCA (EBPA). Average relative deviation from TCL and average improvement (i.e. minimum cost) of the total cost in Table 4 shows that our proposed HCA (EBPA) based technique outperforms GA (BPA) as well as HCA (BPA), while HCA (BPA) out performs GA (BPA).

In Table 4, the relative deviation of each algorithm from TCL, percent improvement of HCA (BPA) over GA (BPA) and HCA (EBPA) over HCA (BPA) are mentioned for each of the utilization factors. HCA (EBPA) has a minimum average deviation of 7.52 from TCL and the results are consistent for both low and high utilization factors. Also, HCA (EBPA) has maximum average improvement of 9.3% over HCA (BPA) and the results are consistent for both low and high utilization factors.

In Table 5, the detailed parameter values used to find the optimum results of ELSP is mentioned. The result consists of fundamental cycle time T, integer multiplier of product K and production position J for each utilization factor.

Fig. 1 depicts the visual representation of the quality of the results obtained through GA (BPA), HCA (BPA) and HCA (EBPA). It is important to note here that the bar graph having minimum height represents the best results because algorithm having minimum deviation from TCL is the best one.

6. CONCLUSION

In this paper we have proposed HCA (EBPA) to solve ELSP on Bomberger's dataset. The results (i.e., relative deviation from TCL and improvement over other algorithms) obtained from the proposed algorithm are better than the existing ones. That the HCA (EBPA) based solution completely outperforms both GA (BPA) and HCA (BPA) for each utilization factor. It is important to mention that the proposed algorithm performed well for both low and high utilization cases. Therefore, it is a significant advantage over other algorithms as most of the algorithms usually failed to find optimum results for high utilization cases.

TABLE 2. DATA OF BOMBERGER'S PROBLEM [20]

Product Index, i	1	2	3	4	5	6	7	8	9	10
Base Demand	24,000	24,000	48,000	96,000	4800	4800	1440	20,400	20,400	24,000
Setup cost (Si): $	15	20	30	10	110	50	310	130	200	5
Production Rate (Pi): units/day	30,000	8000	9500	7500	2000	6000	2400	1300	2000	15,000
Setup time (τi) : h	1	1	2	1	4	2	8	4	6	1
Holding Cost (Hi): $/unit-year	0.00065	0.01775	0.01275	0.01000	0.27850	0.02675	0.15000	0.59000	0.09000	0.00400

TABLE 3. COMPARISON OF TCIS, TCL, GA (BPA), HCA (BPA) AND HCA (EBPA) SOLUTIONS FOR BOMBERGER'S PROBLEM [7-8]

Total Annual Costs							
Utilization (%)	TCIS	TCL	GA (BPA)	HCA (BPA)	HCA(EBPA)	Best Cost	Best Algorithm(s)
50	5960.445	5960.445	6038.410	6032.225	6059.117	6032.225	HCA (BPA)
55	6218.253	6218.253	6328.670	6328.086	6319.254	6319.254	HCA (EBPA)
60	6459.905	6459.905	6621.750	6618.572	6562.772	6562.772	HCA (EBPA)
65	6687.131	6687.131	6914.700	6914.837	6791.523	6791.523	HCA (EBPA)
66.18	6738.810	6738.810	7024.110	7024.100	6843.517	6843.517	HCA (EBPA)
70	6901.335	6901.335	7395.460	7395.460	7006.952	7006.952	HCA (EBPA)
75	7103.674	7103.674	7789.630	7794.202	7210.253	7210.253	HCA (EBPA)
80	7295.114	7295.114	8096.010	8085.485	7402.427	7402.427	HCA (EBPA)
83	7405.090	7405.090	8250.290	8250.290	7512.747	7512.747	HCA (EBPA)
86	7511.593	7511.593	8553.310	8483.945	7619.529	7619.529	HCA (EBPA)
88.24	7588.934	7588.934	8782.420	8782.289	7697.039	7697.039	HCA (EBPA)
89	7614.763	7614.763	8874.550	8874.803	7722.918	7722.918	HCA (EBPA)
92	7714.729	7714.729	9745.800	9746.356	7823.051	7823.051	HCA (EBPA)
95	7811.608	8418.885	12018.080	11949.646	9097.203	9097.203	HCA (EBPA)
97	7874.534	11290.966	17143.000	17134.260	14400.720	14400.720	HCA (EBPA)
98	7905.510	15681.535	24533.820	24457.541	20487.595	20487.595	HCA (EBPA)
99	7936.166	29942.667	55544.470	47550.735	42535.055	42535.055	HCA (EBPA)

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TABLE 4. COMPARISON OF RELATIVE DEVIATION FROM TCL, IMPROVEMENT OF HCA (BPA) OVER GA (BPA), AND IMPROVEMENT OF HCA (BPA) OVER HCA (EBPA) FOR BOMBERGER'S PROBLEM [7-8]

Utilization (%)	% Relative Deviation from TCL			HCA (BPA)	% Improvement CS (EBPA) over CS (BPA)
	GA (BPA)	HCA(BPA)	HCA (EBPA)		HCA (EBPA)
50	1.308	1.204	1.655	0.102	0
55	1.776	1.766	1.624	0.009	0.140
60	2.505	2.456	1.592	0.048	0.843
65	3.403	3.405	1.561	0	1.783
66.18	4.234	4.234	1.554	0	2.571
70	7.160	7.160	1.530	0	5.253
75	9.656	9.721	1.500	0	7.492
80	10.979	10.834	1.471	0.130	8.448
83	11.414	11.414	1.454	0	8.940
86	13.868	12.945	1.437	0.811	10.189
88.24	15.727	15.725	1.425	0.001	12.357
89	16.544	16.547	1.420	0	12.979
92	26.327	26.334	1.404	0	19.734

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TABLE 5. DETAIL RESULTS OF HCA (EBPA) FOR BOMBERGER'S PROBLEM

Utilization	Meta-heuristic
	HCA (EBPA)
50	T = 21.587, K=[8,2,2,1,4,8,16,1,4,2] and J = [3,2,2,1,4,2,16,1,2,1]
55	T = 14.266, K = [16,4,4,2,4,8,16,2,4,4] and J = [1,1,2,1,1,2,2,2,2,3]
60	T = 27.473, K=[8,2,2,1,2,4,8,1,2,2] and J = [5,2,2,1,2,2,5,1,2,2]
65	T = 13.274, K = [16,4,4,2,4,8,16,2,4,4] and J = [1,1,2,2,1,2,1,3,3,3]
66.18	T = 13.173, K=[16,4,4,2,4,8,16,2,4,4] and J = [1,1,2,2,1,2,3,3,3,3]
70	T = 25.730, K=[8,2,2,1,2,4,8,1,2,2] and J = [2,2,1,1,1,2,4,1,2,2]
75	T = 25.006, K=[8,2,2,1,2,4,8,1,2,2] and J = [1,1,1,1,1,2,2,2,2,2]
80	T = 24.358, K=[8,2,2,1,2,4,8,1,2,2] and J = [1,1,1,1,2,2,2,2,2,2]
83	T = 12.000, K=[16,4,4,2,4,8,16,2,4,4] and J = [1,1,2,3,2,3,3,3,3,4]
86	T = 23.663, K=[8,2,2,1,2,4,8,1,2,2] and J = [1,1,1,2,1,2,2,2,2,2]
88.24	T = 23.425, K=[8,2,2,1,2,4,8,1,2,2] and J = [1,1,1,2,1,2,2,2,2,2]
89	T = 23.346, K=[8,2,2,1,2,4,8,1,2,2] and J = [1,1,1,2,1,2,2,2,2,2]
92	T = 11.524, K=[16,4,4,2,4,8,16,2,4,4] and J = [1,2,3,1,2,2,3,4,4,4]
95	T = 40.445, K=[2,1,2,1,2,4,4,1,2,2] and J = [1,1,2,1,2,4,2,1,1,2]
97	T = 81.365, K=[2,1,2,1,2,2,2,1,2,2] and J = [1,1,2,1,2,2,2,1,1,2]
98	T = 138.053, K=[2,1,1,1,2,2,2,1,1,2] and J = [1,1,1,1,2,1,2,1,1,1]
99	T = 262.488, K=[2,1,1,2,2,1,2,1,2,1] and J = [2,1,1,1,2,1,1,1,2,1]

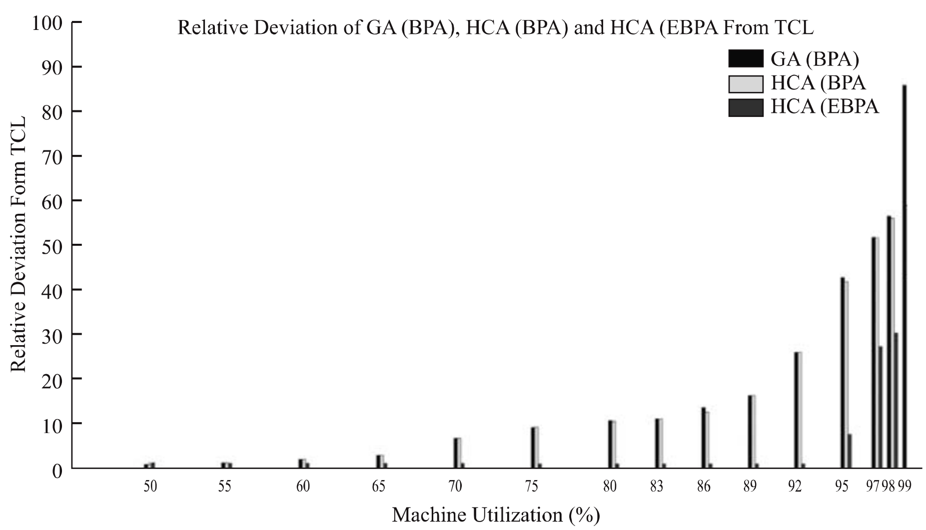

FIG. 1. PERCENTAGE RELATIVE DEVIATION OF GA (BPA), HCA (BPA), AND HCA (EBPA) FROM TCL

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