Steady State Dynamic Operating Behavior of Universal Motor

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ABSTRACT

A detailed investigation of the universal motor is carriedout and used for various dynamic steady state and transient operating conditions of loads. In the investigation, output torque, motor speed, input current, input/output power and efficiency are computed, compared and analyzed for different loads. This paper discusses the steady-state behavior of the universal motor. A non-linear generalized electric machine model of the motor is considered for the analysis. This study was essential to investigate effect of output load on input current, power, speed and efficiency of the motor during operations. Previously such investigation is not known.

Key Words: **Universal Motor, Dynamic Behavior, Steady-State, Transients, Electric Machine Modeling, Simulation**

1. INTRODUCTION

Universal motor is widely used in hand held portable equipments in home, commerce, industry, office and workshop [1,4]. They operate satisfactorily both on AC and DC supply mains. Voltage equations of the universal motor are voltage equations of series circuits of field and armature winding connections, as shown in Fig.1. Using Kirchhof's Loop Law, the differential equations are formed, and then converted into equivalent steady-state equations. The differential equations are nonlinear, difficult and stressful except when solved by numerical integration [1,2]. Characteristics change due to load change on universal motor during steady-state and transient operations [3-5] are studied here. Pavel and Dupej, [6] and Antonino, et. al. [7] used a simple generalized equivalent circuit of armature and field windings. Pavel used harmonic approach in the formation of equations and solved the final equations by MATLAB/Simulink. Antonino suggested more complicated equivalent circuit equation and solved both field and armature circuits by putting the magnetizing resistance across the series combination of field and armature circuits. Okoro [8] used a simple series circuit which met all the aspects of the generalized Machine Theory and solved the equations by Cramer rule. His work is not suitable, as the results are non-converging. Haci and Bodur [9] obtained their results through Microcontroller and their final equations are not clear. Their results show that they had used sampling approach. The research work presented by Mohammad and Widyan [10] used generalized machine equations, and solved them by MATLAB math toolbox.

In this paper, steady-state results are obtained by two different procedures. In the first method a complete set of differential equations for the universal motor are

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formed and solved by Runge-Kutta integration for some constant output loads. After transient period dies down, the currents, voltages, input/output powers etc. become oscillatory sinusoidal periodic steady-state for ac. voltage supply and constant for dc voltage supply. In the second method steady-state results are obtained from the dynamic steady state solution. The output dynamic results are graphically presented, compared, discussed and concluded.

2. FORMATION OF UNIVERSAL MOTOR CIRCUIT

This paper uses basic series non-linear generalized machine equivalent circuit shown in Fig. 1. The exact equivalent circuit of the field and armature windings of the universal motor is similar to a transformer equivalent circuit, with modification that the mutual inductance of the motor circuit is variable due to position change of armature wrt the field [11,12].

3. FORMATION OF BASIC ELECTRICAL MACHINE EQUATIONS

The universal motor transient voltage and armature torque equations are based upon series connection of field and armature circuits of Fig. 1, and in matrix form the differential equations are [11,12]:

$$\begin{bmatrix} v^f \\ v_a \end{bmatrix} = \begin{bmatrix} R_f & 0 \\ 0 & R_a \end{bmatrix}\begin{bmatrix} i_f \\ i_a \end{bmatrix} + \begin{bmatrix} L^f_{Self} & 0 \\ 0 & L^a_{Self} \end{bmatrix}\frac{d}{dt}\begin{bmatrix} i_f \\ i_a \end{bmatrix} + \omega\begin{bmatrix} 0 & 0 \\ M_m^{fa} & 0 \end{bmatrix}\begin{bmatrix} i_f \\ i_a \end{bmatrix} \tag{1}$$

$$T_e = \frac{pp}{2}\begin{bmatrix} i_f & i_a \end{bmatrix}\begin{bmatrix} 0 & 0 \\ M_m^{fa} & 0 \end{bmatrix}\begin{bmatrix} i_f \\ i_a \end{bmatrix} = ppM_m^{fa} i_f i_a = ppM_m^{fa} i^2 \tag{2}$$

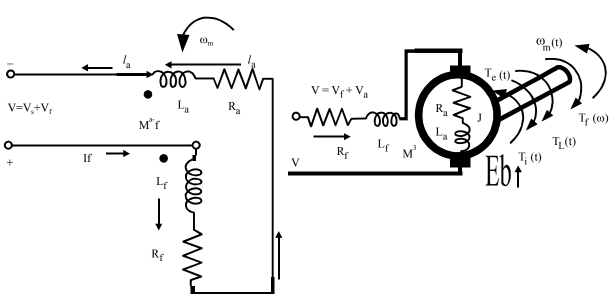

FIG. 1. GENERALIZED/UNIFIED EQUIVALENT CIRCUIT (ALONG WITH MECHANICAL TORQUES) OF THE UNIVERSAL MOTOR

where pp is pair of poles in the motor. The 2nd order non-linear terms ωi_f in Equation (1) and i_f, i_a in Equation (2) make the voltage and torque equations nonlinear of the 2nd order. For the series connected field and armature windings, total single supply voltage equation from Fig. 1 is:

$$v = (R_f + R_a)i + (L^f_{Self} + L^a_{Self})\frac{di}{dt} + \omega M_m^{fa} i \tag{3}$$

After a reasonable long time, the transient operation of Equation (3) becomes steady state Equation (4) as given:

$$V = (R_f + R_a)\, I + j\{(L^f_{Self} + L^a_{Self})2\pi f\} I + \omega M_m^{fa} I \tag{4}$$

or

$$V = (R_a + R_f)\, I + j(X^a_{Self} + X^f_{Self}) I + \omega M_m^{fa} I \tag{5}$$

For the dc steady state voltage supply, Equation (3) or Equation (5) becomes:

$$V = (R_f + R_a)I + \omega M_m^{fse\,a}\, I \tag{6}$$

The torque equation for ac and dc voltage are given by:

$$T_e = pp\, M_m^{fa}\, i^2 = pp\, M_m^{fa}\, I^2 \tag{7}$$

Equation (7) shows that ac torque is variable positive for +ve and -ve halves of current frequency and the torque has the double frequency of that of supply and sinusoidal.

4. FORMATION OF DYNAMIC TRANSIENT EQUATIONS

Since armature torque T_e produced overcomes load torque T_L, friction torque T_F and acceleration $J\frac{d}{dt}\omega_m$, the dynamic torque balance and speed balance Equations (8), as shown in Fig.1, are:

$$T_e = J_m \frac{d}{dt}\frac{\omega}{pp} + T_L + T_f(\omega) \tag{8}$$

or

$$T_T = T_e - T_f - T_L = \frac{J_m\, d\omega}{pp\ dt} \tag{9}$$

and

$$0 = \frac{d\theta}{dt} - \omega \tag{10}$$

Equations (3,9,10), in matrix form provides the complete electro-dynamic numerical integration equations as given:

$$\begin{bmatrix} v \\ T_T \\ 0 \end{bmatrix} = \begin{bmatrix} R_f + R_a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} + \begin{bmatrix} L_f + L^a_{Self\text{-}\max} & 0 & 0 \\ 0 & \frac{J}{pp} & 0 \\ 0 & 0 & 1 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} + \omega \begin{bmatrix} M^{fa}_m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} \tag{11}$$

Solution of numerical Equation (11) is:

$$\frac{d}{dt}[i] = [L]^{-1}\{[v] - [R][i] - \omega[G][i]\}$$

or

$$\frac{d}{dt}\begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} = \begin{bmatrix} L_f + L^a_{Self} & 0 & 0 \\ 0 & \frac{J}{pp} & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \left\{ \begin{bmatrix} v \\ T_T \\ 0 \end{bmatrix} - \begin{bmatrix} R_f + R_a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} - \omega \begin{bmatrix} M^{fa}_m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i \\ \omega \\ \theta \end{bmatrix} \right\} \tag{12}$$

where $T_e=(pp)M^{fa}_m\ i^2$,v,i, L^a_{Self} and R_a are armature generated torque, applied voltage, armature current, armature self-inductance and armature winding resistance respectively.ω,M^{fa}_m i, are speed, mutual inductance and back emf respectively.

5. FORMATION AND EXECUTION OF DYNAMIC STEADY-STATE EQUATIONS

The steady-state computer program starts from first assumed armature torque T_e=16.5 in the first run and then an increase of 2.5 N-m in each next run, till the maximum output of the motor is achieved. Armature current from Equation (2) at each armature torque is:

$$I = \sqrt{\frac{T_e}{pp\ M^{fa}_m}} \tag{13}$$

This current I is dc amperes for dc supply and rms amperes current for ac supply. W is calculated from ac supply voltage by Equation (5) multiplied by I and for dc supply voltage by Equation (6) multiplied by I. Angular speed ω is calculated from Equation (5) rearr-anged as Equation (14):

$$\left(\frac{V}{I}\right)^2 = \left(R_{eq} + \omega M^{fa}_m\right)^2 + \left(X_{eq}\right)^2$$

or

$$\omega = \frac{1}{M^{fa}_m}\left[\sqrt{\left\{\left(\left(\frac{V}{I}\right)^2 - \left(X_a + X_f\right)^2\right)\right\}} - \left(R_a + R_f\right)\right] \tag{14}$$

The speed ω can easily be converted into speed N_{rpm} by $N_{rps}=\omega/2\pi$ and $N_{rpm}= \omega/120\pi$ or $N_{rpm}=60N_{rps}$. At this stage, ω is inserted into friction torque $T_f=k_f\,\omega^2$. Considering the fact that an steady-state operation of an electrical machine gives only one value of current, speed, torque, efficiency etc. the computer program is calculated exactly under correct values at given magnitudes of load torque and applied voltage as under:

$$\begin{aligned} &T_T = T_e - T_L - T_{fric} = T_{inertia} \\ &E_b = \omega\, M^{fa}_m\, I \\ &P_{out} = E_b\, I - \omega T_f - \omega T_L \\ &P_{in} = V\ I\ pf \\ &P_{arm} = E_b\, I \\ &\eta = \frac{P_{out}}{P_{in}} = \frac{\omega T_L}{V\ I\ pf} \end{aligned} \tag{15}$$

$$\begin{aligned} &T_T = T_e - T_f - T_L = \frac{J_m\, d\omega}{pp\ dt}, \\ &pf = \frac{\left(R_a + R_f + \omega M^{fa}_m\right)}{\sqrt{\left\{\left(R_a + R_f + \omega M^{fa}_m\right)^2 + \left(X_a + X_f\right)^2\right\}}} \end{aligned} \tag{16}$$

For the dc voltage supply, Equation (16) will become 0, and substituting Zero for frequency f=0 , it is also zero. The steady-state program is in fact execution of Equations (12-14) in the above given sequence.

6. STEADY-STATE RESULTS FROM STEADY-STATE PROGRAM

Figs. 2-3 compares conventional steady-state program results for ac and dc steady-state efficiency e_{fc}, power factor p_f, Input power P_{in}, feedback voltage E_b, armature power P_{arm}, Mechanical power output P_{out}, Input current I, armature torque T_{orq}, load torque T_{load} against speed ω rad/s. From characteristic curves, the motor performance is at high efficiency and high p_f between speed ω=20 radians per second to speed ω=30 radians per second. Power out-put in this range varies between $60 \times 29 = 1450$ watts to $60 \times 29 = 1740$ watts. This means the motor rating is approximately 2-2.5 h_p. On the dc supply, pf curve of the motor is unity. High dc efficiency remains in the same speed range of 20-30 rad/sec. Conventional circuit ac and dc voltage steady-state results are of almost same shape, smooth, average results for the series dc motor.

Fig. 4 compares ac and dc feedback voltage, p_f, and current against speed in rad/s along x-axis. DC values (solid lines) are either at the same magnitude or at a little higher value than the ac quantities (dotted lines) for the same speed rad/sec. These results show that the ac and dc voltage performance by conventional solution are of almost same shape, nature and magnitude.

Fig. 5 compares universal motor performance on ac and dc supply for efficiency, output torque and load torque against current amperes along x-axis. Fig. 5 shows efficiency and torque versus input current in amperes. The maximum efficiency is 85% between 9 amperes and 11 amperes and average high efficiency range starts from 8-20 amperes. Dc voltage efficiency is relatively same and a little higher than the ac voltage efficiency.

Fig. 6 compares ac and dc feedback voltage, p_f, and speed against current in amperes along x-axis. These curves show that efficiency and power factor of the universal motor are little higher for dc supply than these values for ac supply. The curves in these graphs also show that ac power supply efficiency is slightly lower than dc power supply efficiency of the motor. The curves show that the load torque is less than armature torque by the difference of friction torque. These curves show that for the same current, dc voltage efficiency and dc torque output of the motor are higher than the same quantities with the ac supply. And the performance of the Universal motor under dc supply is a little superior than its performance with ac supply.

7. AC AND DC VOLTAGE STEADY-STATE BEHAVIOR FROM TRANSIENTS

Steady-state behavior of the universal motor is also obtained from transient program after transients have died down. If dc voltage is applied the steady state, results are constant and if ac voltage is applied, the result is oscillatory. Since transient program calculates and shows results in instantaneous values, ac steady-state current, flux, voltage, torque, power and efficiency etc. from the program are instantaneous and when plotted the steady state results look sinusoidal. Time is linear in transients and therefore it is taken as reference in plotting transient quantities and steady state quantities. The steady-state armature torque of the universal motor T_e=pp M^{fa}_m I^2 is of double supply frequency and of square positive unidirectional magnitude pulsating sinusoidal. The ac and dc voltage steady-state behaviors obtained from input and output powers of the transient program are shown in Fig. 7.

Fig. 7 compares ac and dc input power flow into the universal motor during steady state operations. The power drawn by the motor from ac source is positive and sinusoidal except a very small negative power dip at zero crossing of the current due to small out of phase voltage and current (or small out of phase fluxes on rotor and stator at higher load). It shows that reversal of the current is not exactly at the same time as the reversal of voltage. This small negative power dip appears only at high load outputs. Higher the output load, higher is the negative dip and also higher is the positive maximum peak load. With constant output load torque from the universal motor, the power input from the dc source supply is constant and always positive. Higher the constant output load, higher but constant is the power intake from the dc supply. Fig. 7 shows input powers from ac and dc power supplies. These curves compare performance of the universal motor when the power is supplied from dc and ac

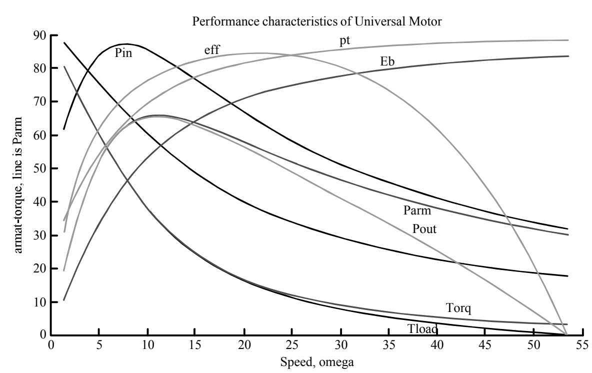

FIG. 2. GRAPH BETWEEN AC PIN VS SPEED, EFF VS SPEED, PF VS SPEED, EB VS SPEED, PRM VS SPEED, POUT VS SPEED

power supplies for a constant output load torque. Definitely for a constant output load torque and with ac supply, generated armature power is sinusoidal and with the dc supply, the armature torque and input power are constant. Higher the output load, higher is the power input to the universal motor and vice versa.

Fig. 8 shows final steady-state speed of universal motor gained on 200V ac and 200V dc voltage supply at fixed output loads 50, 25 and 0 N-m. Speed of the universal motor at some constant load output and dc supply is higher than its speed at the same output load with ac supply. Also lower the output load torque, higher is the speed and vice versa. The graph shows that the universal motor has highest speed at no load and low speed at high output load. For the constant output load and with the ac source, the motor has oscillatory periodic variable speed whereas with constant output load and with dc source, the motor has constant speed. For the same supply voltage, higher is the mechanical power output, lower is the speed of the universal motor and vice versa.

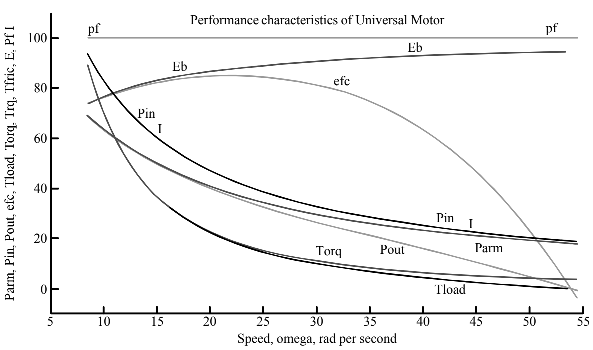

FIG. 3. GRAPH BETWEEN DC PIN/EPEED, EFF/SPEED, PF/SPEED, EB/SPEED, PRM/SPEED, TLOAD/SPEED

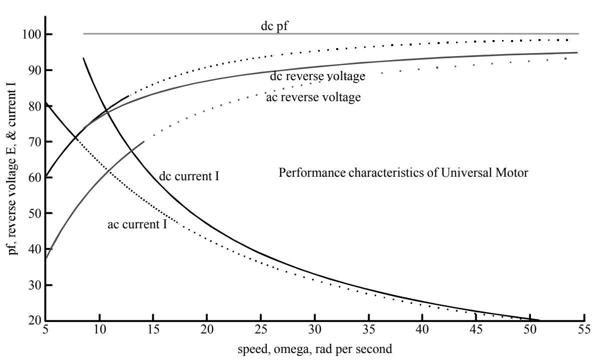

FIG. 4. SHOWS GRAPH BETWEEN PF/SPEED, CURRENT/SPEED, EB REVERSE VOLTAGE/SPEED FOR DC (---) AND AC(……)

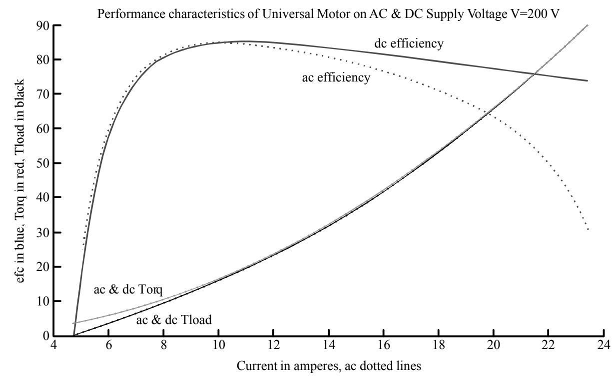

FIG. 5. GRAPH BETWEEN EFFICIENCY VERSUS CURRENT AND TORQUE VERSUS CURRENT FOR DC (----) AND C (…..)

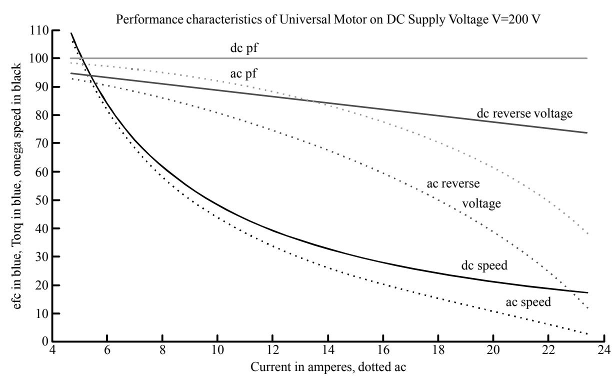

FIG. 6. GRAPH BETWEEN PF VERSUS CURRENT , EB VERSUS CURRENT AND SPEED VERSUSCURRENT FOR DC (---) AND AC (…..)

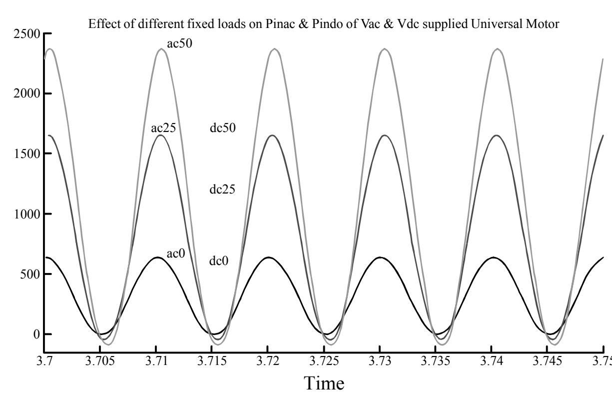

FIG. 7. GRAPH BETWEEN PIN VERSUS TIME AT DIFFERENT LOADS FOR DC (HORIZONTAL ST LINES) AND AC (SINUSOIDAL)

Figs. 9-10 compares armature torque in N-m for constant values of output torques for ac and dc voltages. Although the output load torques are constant, with an ac supply, the armature torque T_a=pp $M^{fa}{}_m$ i^2 is variable due to variable nature of ac current and speed of the universal motor with ac current. The armature torque is constant with constant output load torque and dc voltage source. AC armature torque developed varies oscillatory sinusoidal from 0 N-m to some constant maximum peak. With constant output motor load torque, and with ac supply, the ac armature torque developed varies oscillatory periodically sinusoidal between 0 N-m and a constant maximum peak value. With dc source and constant output load torque, dc armature torque developed is a constant magnitude. During steady-state operation with ac voltage supply, armature torque is oscillatory sinusoidal variable, irrespective of the shape of the motor output load torque, also the inertia torque is sinusoidal nearly symmetrical across the zero axis along time axis. During steady-state operation of universal motor with dc supply voltage, armature torque is sinusoidal for sinusoidal output load torque and the armature torque is constant for constant output load torque with the inertia torque equal to near zero friction losses. Armature torque is higher for higher output load torques and vice versa.

During steady state operation with ac voltage supply, inertial torque is oscillatory sinusoidal nearly symmetrical over zero torque axis in the direction of time increase. Steady state inertia torque is zero for dc supply voltage and constant output torque. Higher the output load torque, higher is the inertia torque with ac voltage supply.

At all times, with the ac voltage supply to universal motor, steady-state generated torque is equal to friction torque plus load torque plus inertia torque. At any moment, with the dc voltage supply to the universal motor steady state generated torque is equal to friction torque plus load torque plus zero inertia torque. Inertia torque with dc supply during steady-state operation is zero. With ac supply during steady-state operation, generated torque is equal to sum of load, friction and inertia torques. With dc supply during steady-state operation, generated torque is equal to sum of the load torque and friction torque and zero inertia torque because, inertia torque is equal to zero with dc supply during steady state operation. Steady state Inertia torque with ac supply and for any output load torque is sinusoidal periodic oscillatory. There is zero inertial torque with dc supply. Due to variable nature of inertia torque, the steady-state speed of the universal motor with ac voltage supply is also variable. Due to zero inertial torque with dc voltage during steady-state, the speed of universal motor is constant. These curves also show that during dc supply steady-state operation of

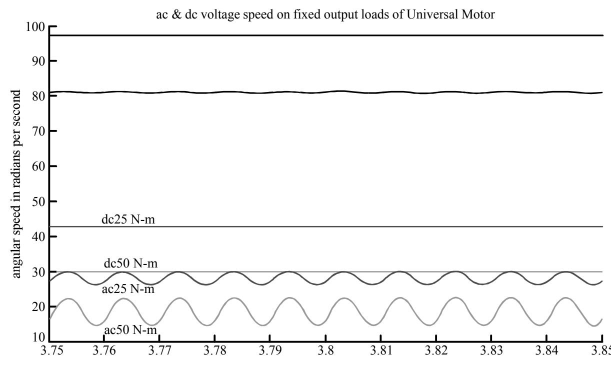

FIG. 8. SHOWS GRAPH BETWEEN SPEED/TIME FOR VARIOUS LOADS FOR AC & DC VOLTAGES AS DC (HORIZONTAL STRAIGHT LINE) AND AC (HORIZONTAL SINUSOIDAL LINES)

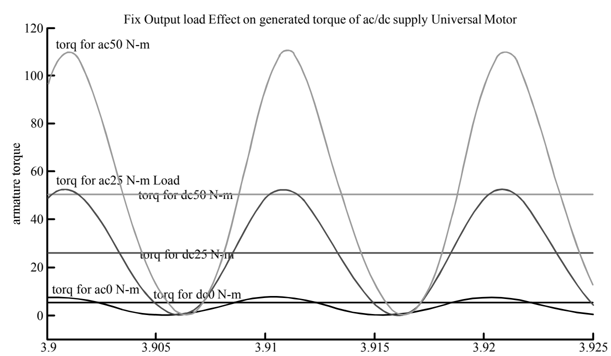

FIG. 9. GRAPH BETWEEN ARMATURE TORQUE/TIME AT VARIOUS LOADS FOR DC (CONSTANT HORIZONTAL) AND AC (SINUSOIDAL)

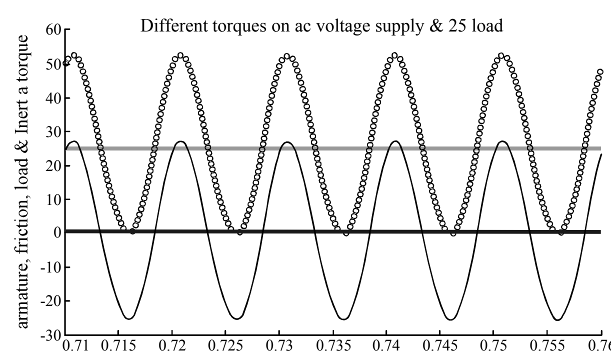

FIG. 10. GRAPH BETWEEN INERTIA TORQUE/TIME AT VARIOUS LOADS FOR DC (CONSTANT HORIZONTAL) AND AC (SINUSOIDAL)

Universal motor the total armature torque is equal to the frictional loss torque plus output load torque. Since ac supply produces variable armature torque, and also there is variable inertial torque, the input power to the universal motor is variable. The inertial torque provides the motor variable or fluctuating speed. The inertial torque and hence the variable speed are periodic in magnitude, and therefore the performance speed, current and power input and output of a universal motor are periodic steady-state. Note that the armature, output, friction and inertia torques are of the constant magnitude with dc supply and variable with ac supply. There is also periodic oscillatory inertial torque, symmetrical on both sides of zero torque axis in the direction of time increase with the ac voltage supply due to variable nature of armature torque. With the ac voltage supply, inertial torque is periodic and produces periodic motor speed.

8. CONCLUSION

In this study the generalized circuit of the universal motor is developed and its circuit equations derived and described for the dynamic connections of field and armature coils in series. The electro-dynamic/electro-mechanical differential equations are programmed for dynamic steady state and transient solutions, and the programming was carried out successfully and effectively. The program results are shown and analyzed and printed in this paper. In this paper a detailed study has been made for dynamic steady-state performance with ac and dc voltage supplies using conventional steady-state and transient generalized electrical machine approaches for steady-state performance of the universal motor. With dc supply steady-state results from conventional analysis are similar in nature and shape in generalized machine transient analysis. However steady-state analysis with ac supply to universal motor, the steady state torque, current, speed, power etc. from generalized machine transient approach are oscillatory periodic sinusoidal unlike the smooth average dc voltage like results from conventional approach method. The difference between these results are very clearly shown and defined for the first time. The analysis of the universal motor confirms the work and confirms that the steady-state analysis can be done on the basis of transient differential equations and programming. Their ac and dc transient and steady-state solution analysis curves of other authors are similar to the dc steady-state solutions determined in this paper. It was further observed that when the moment of inertias of armature and drive are large enough, the torque oscillations were reduced in rotational speed. The ac voltage rotational torque-speed and torque-time responses with high inertia loads are nearly similar smooth series dc motor torque-time characteristics, in that case the universal motor has the same rotational speed-torque characteristics as that of the series-wound dc machine fed from DC source.

9. SUGGESTIONS FOR FUTURE WORK

(i) Dynamic behavior of universal motor against variable load of water pump-set be determined.

(ii) Dynamic behavior of the universal motor against grass cutting load be examined.

(iii) Dynamic behavior of compensated winding universal motor with and without the winding be determined.

(iv) Dynamic behavior of inter-poled Universal motor with and without inter-pole be investigated.

(v) Complete dynamic speed control of universal motor by resistance, reactance, field diversion, pole tapping of the universal motor be investigated.

(vi) Study of the universal motor under various sudden load changes and sudden inertia changes after switching be investigated.

(vii) Study of the universal motor with gear, translational and direct load and sudden change of the load and the inertia be investigated.

(viii) Study of Electronic control and ac drive be made and studied in detail.

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REFERENCES

[1] Mobolaji, B., and Davison, I., "Dynamics of a Solar Powered Fractional Horse Power Motor", 2nd International Conference on Electrical & Electronics Engineering, ICEEE & XI Conference on Electrical

Engineering, pp. 273-277, Mexico City, Mexico, September 7-9, 2005.

[2] Aharon, Y., and Appelbaum, J., "Transient Analysis of a dc Series Motor (Linear Versus Nonlinear Models)", IEEE Transactions on Industrial Electronics & Control Instruments, Volume 28, No. 2, May, 1981.

[3] Zein-El-Din, A.S., El-Shebiny, M.E., and Khater, M.M., "Micro Computer-Controlled Universal Motor", Proceedings of IEEE International Symposium of Electrons, pp. 653-658, 1996.

[4] Zein El Din,A.S., S.A. Mahmood, "Efficient microcontroller-based universal motor", MCON'94, IEEE International Electronics Society in Technical-sponsorship, Bologn, Italy, Sept 5-9, 1994

[5] Slemon, G.R., and Straughen, A., "Electric Machines", Addison Wesely Publishing Company, New York, 1982.

[6] Pavel, Z., and Dupej, J., "Modeling of an Universal Motor Supplied by a Harmonic Voltage", Power Electronics and Motor Control Conference, pp. 1070-1073, Portoroz, Slovenia, 2006.

[7] DiGerlando, A., Perini, R., and Rapi, G., "Equivalent Circuit for the Performance Analysis of Universal Motors", IEEE Transactions on Energy Conversion, Volume 19, No.1, pp. 18-27, March, 2004.

[8] Okoro, O.I., "Application of Numerical software in Electrical Machine Modeling", Journal of Engineering & Technology, 2005.

[9] Mamora, K.K., "Commutation Analysis for High Speed Universal Motor", Compumag, Browse Sections Conference, Corvina, July, 2013.

[10] Pankaj, S., and Pandey, A.K., "Modelling Open & Closed Loop Response of Universal Motor", International Journal of Electrical, Electronics & Communication Engineering, Volume 3, No. 5, pp. 225-228, May, 2013.

[11] Gupta, R., Lamba, R., and Padhee, S., "Thyristor Based Speed Control Techniques of DC Motor: A Comparative Analysis", International Journal of Scientific & Research publications, Volume 2, No. 6, pp. 1-6, June, 2012, www.ijsrp.org

[12] Pankaj, S., and Pandey, A.K., "A Comparative Analysis Between Fuzzy Controller & PID Controller for a Universal Motor", Asian Journal of Engineering Research, Volume 1, No. 4, pp. 1-4, July-September, 2013.

[13] Haci, B.A. Bakan, F.M. and Sarul, H., "Universal Motor Speed Control with Current Controlled PWM AC Chopper by Using a Microcontroller", Proceedings of IEEE International Conference on Industrial Technology, pp. 394-398, 2004.

[14] Widyan, M.S., Anas, I. Al-Tarabsheh, Issa, Y.E., and Rolf, E.H., "Transient Analysis and Output Characteristics of DC Motors Fed by Photovoltaic Systems", Jordon Journal of Mechanical and Industrial Engineering, Volume 4, No.1, pp. 193-204, January, 2010.

[15] Khan, M.R., Ahmad, S., Azam, M.M., Hussain, S.K., and Khan, F., "Analysis & Controller Design of a Torsional DC Drive under Resonance Conditions", Universal Journal of Electrical & Electronic Engineering, Volume 1, No. 2, pp. 41-51, 2013, http://www.hrpub

[16] Burdi, M.K., "Steady-State and Transient Behavior of a Double Cage Induction Motor", M.Sc. Thesis, Loughborough University of Technology, UK, 1980.

[17] Burdi, M.K., "Class Notes in the Subject of Advance Electrical Machines", University College of Engineering & Technology, Islamia University, Bahawalpur, Pakistan.